The Mathematics of Popcorn

March 30, 2010

A kernel of popping corn consists of a hard outer shell called the pericarp which is impermeable to moisture and a dense center filled with starch, water and some oil[8]. As the kernel is heated, internal pressure rises to approximately 135 psi, at which point the pericarp bursts. The steam and starch expands to a foam which rapidly cools forming the popcorn. Variations in the strength of the pericarp, mass and composition of the core, heating temperature and other factors contribute to the time required to pop the kernel. In controlled experiments, Byrd and Perona[2] found that the number of unpopped kernels remaining after time t is described by

$$N = N_0 e^{-kt}$$

where N_0 is the number of unpopped kernels initially. The model they used consists of a system of differential equations, the first being Newton's Law of Cooling, and the second describes the rate of popping:

$$\frac{dT}{dt} = c\,(T_a - T) \tag{1}$$

$$\frac{dN}{dt} = -kN. \tag{2}$$

T_a is the ambient temperature the surrounding air or of the oil in which the popcorn is heated. The distribution of critical pressures was assumed to be Gaussian. The website Mixed Apparatus for Radar Investigation of Atmospheric Cosmic-rays of High Ionization (MARIACHI)[5] describes a laboratory procedure to measure the times when individual kernels pop. Byrd and Perona carefully controlled their experiments by popping only one kernel at a time, while the MARIACHI setup uses a more traditional method that heats all of the kernels simultaneously and in close proximity. It has been suggested by others that there are several mechanisms driving the time at which an individual kernel pops. The first is that the pressure in the kernel builds over time, but with an unknown variance which we wish to determine. Secondly, the probability of popping (failure of the pericarp) increases as a function of time, and finally there are

variations in the burst pressure of the pericarp itself. A revised model is given by

$$N(t,T) = 1 - \frac{\exp\left(\frac{-\beta}{f(t,T)}\right)}{\left(1 + \frac{\alpha}{f(t,T)}\right)} \tag{3}$$

$$f\left(t,T\right) = \exp\left(R(t,T)\right) - \exp\left(R(0,T)\right) \tag{4}$$

$$R(t,T) = k\left(T - T_c\right)^2 t - c\left(T - T_c\right) \tag{5}$$

Equation 4 represents the accelerating probability that the pericarp will burst as the stress increases. Below the critical temperature, T_c, no kernel will pop so the rate function $R(t,T)$ goes to zero. Whether this is the correct model for the distribution of the times that popcorn kernels pop remains to be determined through experiments, but it seems likely that a fat tail distribution is a better fit than the normal distribution[6]. Other applications of fat tailed distributions are found in sizes of cities[3], walking patterns[7] and the distribution of the sizes of oil fields[1]. The rate of discovery of new oil fields is of particular importance as it relates to the economies of the world's nations[4]. The problem consists of data collection, model building and parameter estimation.

Equipment required:

- Microwave popcorn and oven
- Microphone and Audacity software
- Modeling and analysis software: Matlab, Scilab, R, Octave, Vensim or similar

References

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