Dynamic Modeling of Behavior Change in Problem Drinkers

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Abstract

We present a novel dynamical systems modeling approach to understand the processes governing an individual’s behavior in the context of problem drinking. Recent advances in technology have resulted in large intensive longitudinal data sets which are particularly well suited for study within such frameworks. However, the lack of previous work in this area (specifically, on the inter- and intra-personal factors governing drinking behavior in individuals) renders this a daunting and unique challenge. As a result, issues which are typically routine in mathematical modeling require considerable effort such as the determination of key quantities of interest, and the timescale on which to represent them. We discuss the construction of an initial mathematical model for two starkly distinct individuals and make a case for the potential for such efforts to help in understanding the underlying mechanisms responsible for behavior change in problem drinkers.

1 Introduction

In the study of drug and alcohol abuse, there has been extensive information collected on substance use, participant’s willingness to change behavior, and participant’s success in a particular treatment. Field experts have formulated various ideas concerning factors that control a patient’s motivations and behavior. However, the relative contributions of these driving factors, and specifically the mechanisms for behavior change, are unclear. These interacting factors, which are inherently then complex and nonlinear, change over time and it is natural to explore these issues within the framework of a mathematical model, which enables us to describe these processes quantitatively as a dynamical system. Automated data collection systems have enabled the collection of large data sets involving both high numbers of patients as well as time points (earning the term ‘intensive longitudinal data sets’). We seek to model drinking behavior and associated behavior change as informed by such a dataset, Project MOTION, to gain insight into the underlying mechanisms.
Certainly, the application of mathematical modeling to understand learning processes and social behavior is not a new field and there have been fruitful efforts in the past. Briefly, but not exhaustively, some relevant works include those of Frey and Lau [15] in which a system of integral equations was used to describing ways in which governments make decisions. Van Geert for example in [39] and [40], employs technically diverse tools in the form of a logistical model and a cellular automaton model to understand cognitive development of children. Notably, the work of Stephen Grossberg, summarized nicely in [18] and [19] is a significant contribution to the theory of learning in the context of cognitive mechanisms as well as associated neural network dynamics.

We note that there has been other recent work resulting from the collaborations of field experts in substance abuse and mathematicians. These have been largely at the ‘population-level’, or specifically, the characterization of individuals according to their use (‘heavy’, ‘light’, ‘abstaining’, etc.) similar to epidemiological or ecological models. The goal is then to investigate which scenarios result in overall less or more drinking/substance use in the population. Some contributions along these lines are that of Scribner et al., [36] and Ackleh et al., [1], which focus on student populations and exploring campus alcohol policies with the goal of reducing ‘wetness’. This work was further extended in Rasul et al. [32], where the model was calibrated using parameter estimation and specific alcohol policy scenarios were explored.

Another series of efforts exploits the ideas of theoretical epidemiology. In 2004, Gorman et al., [16] outlined the ways in which studies on alcohol could be improved via dynamic systems modeling and control theory. This note came out of a meeting on Ecological Modeling of Alcohol-Related Behavior sponsored by the NIAAA and possibly began the collaboration that gave rise to a series of papers. Sanchez’s initial paper [34], classifying individuals as susceptible (nondrinkers, S), drinkers (D), and recovered alcohol users (R) has the same structure as a typical SIR model. The difference is that the relapse term, (recruitment of R to D) is modeled by an interaction term, giving rise to a backward bifurcation. This implies that recovered alcohol users can easily relapse through even few social interactions with other drinkers. This work was extended in [12], to explore the role of nonlinear relapse among networks of drinking communities with varying connectivities. Mubayi (in [30, 29]) extends this work to examine the contagion of drinking (using the same basic ‘SDR’ principles), amongst individuals in low- and high-risk communities, and explored the role of residence times in each community. Also, it is shown that social contacts or extent of mixing within communities drastically affects the outcomes. In [30], the deterministic model was extended to consider variability in social interactions of drinkers, and increasing/decreasing levels of drinking. Then the distribution of drinking levels under prevention, intervention, and a combination of both was presented and discussed.

Research on substance use over the past several decades has attempted to identify predictors of successful achievement of abstinence and moderation. There are four sets of categories or constructs that have been broadly identified as being related to sustaining the status quo or prompting changes in drinking behavior. These categories are: 1) stable characteristics of the patient, 2) mood and affect, 3) environmental factors, such as social networks and stressful events, and 4) internal process factors, such as motivation to change, commitment to changing, self-efficacy, etc.

Stable and/or personal characteristics of individuals have been known to affect drinking outcomes. For example, age, gender, and history of a drinking problem are all known and established predictors of both natural recovery and treatment outcomes [11, 24, 22, 37]. Both positive and negative mood and/or affect and how individuals tolerate and react to them, have been associated with maintenance of drinking behaviors. Environmental factors, such as one’s social networks, which can be supportive of continued substance use or supportive of reduced or cessation of substance use, are known to be highly influential in individuals’ decision making processes, access to drugs and alcohol, and recovery itself, such as in the case of Alcoholics Anonymous and other community,
peer-led recovery groups [17, 26, 27]. Finally, there are a number of internal process factors that help to facilitate or impede changes in ones drinking. These factors are usually the primary targets for behavioral treatments. Examples of such factors are motivation to change, readiness to change, commitment to changing ones drinking, self-efficacy, and urges/craving for alcohol.

The MOTION data set is comprised of information collected via an interactive voice recording (IVR) from drinkers who wanted to reduce their drinking, although not necessarily cease drinking altogether. Ideally, each individual answered this 41-question survey once per day for eight weeks, or 56 days, and gave information about inter- and intrapersonal factors that could potentially influence their drinking as well as their drinking behavior in the past 24 hrs and their commitment to avoid heavy drinking or drinking at all in the next day.

Within Project MOTION, both the IVR-based daily survey and the fixed assessments attempted to collect data across these broad categories in order to inform the process of change for individuals aiming to moderate their drinking. These data were collected from participants both within the context of a brief treatment and through independent self-monitoring. Data yielded from each of these measures and individual items were utilized to help create an informed picture or guide to how individuals attempt to change their drinking and how the trajectories of change may differ across groups.

Generally, the use of mathematical modeling to learn about a physical or biological process is done as an iterative process in which an initial model is formulated and then compared with observed or experimental data. This comparison gives insight into any discrepancies between the observed processes and model predictions, suggesting refinements to the model. This process is repeated until the model provides sufficient information to capture key aspects of behavior of the observed system and to answer questions of interest.

However, in most applications where previous models have been developed and/or there is previous knowledge of quantitative behavior of underlying processes, modeling is typically initiated with an idea of the appropriate basic framework and possible mechanisms. This is not the case for this particular problem, and the modeling here is with the goal of working toward such a framework. As such, model terms representative of possible mechanisms are considered on the basis of how well the model solutions then reproduce the observed trends in data. Only after several such efforts are successful are the use of more precise inverse problem methods appropriate as they rely on the mathematical model being a relatively good description of the observed processes. Currently, there are also some challenges in the observation process, or measurements, that would impede the application of such methods.

While the aim of the current work was simply to attempt to develop a mathematical model that may be able to aid in our understanding of any behavioral mechanisms relevant to drinking behavior, the potential is far-reaching. A cohesive theory of how and why individuals change as they modify their behavior long-term is lacking. It is not expected that all mechanisms will be at work in all individuals as there is considerable variability, but there are likely groups or cohorts of people who are motivated and change similarly. Working towards a mechanism-based description of behavior change, these and similar efforts could potentially help clinicians identify patients as part of a characterized cohort and therefore to employ more effective therapies.

Secondary to this goal was the aim of improving upon current data collection standards and study design, which can greatly improve our ability to learn about these processes and make them more amenable to mathematical approaches. As we will discuss, the development of these first two initial models has been enlightening, and some immediate changes became clear. Further changes, representing somewhat of a paradigm shift are discussed in the closing remarks of this paper.

There is no straightforward way of organizing data on such a large variety of aspects of daily life into a reasonable number of variables to study within a mathematical modeling framework.
Further complicating this issue is the lack of previous modeling efforts in this area, resulting in a lack of pertinent quantities commonly agreed upon. To begin formulating conceptual variables, we applied linear methods such as calculating coefficients of determination, linear interpolation via least-squares approximation and principal component analysis. However, these methods do not provide any useful information on relationships between an individual’s responses to questions. It is sensible to construct our model variables on questions based on similar ideas used in the IVR. We shall examine how these categories relate to each other, and how to best use this data in formulating a model. Our initial interpretation of this data is based on knowledge and predictions from the field of substance abuse and recovery, visually assessing qualitative trends between similar questions within topical categories, and grouping them into variables. Once variables were decided upon, qualitative trends between them were examined to determine the existence and nature of relationships between them within a single patient.

We construct this first model based on prior knowledge of hypothesized relationships between the model variables based on knowledge of substance abuse therapy and relationships observed in the data from Project MOTION. We seek to construct a model such that solutions agree with our prior knowledge and are similar to the data. The processes involve nonlinear and delayed components contributing to decision-making and behavior modification mechanisms. This initial model is then compared to data of selected patients, and further refinements and next steps are discussed.

2 Project MOTION: Rationale, Procedures, and Data

Motivational interviewing (MI) has been demonstrated to be an effective stand-alone intervention for alcohol use disorders (AUD). The consistency, magnitude, and durability of its effects, especially given its brevity, suggest powerful mechanisms of behavior change (MOBC) are operating to reduce drinking. Thus, gaining a better understanding of the underlying MOBC in MI is important. However, existing MOBC studies of MI have yielded limited and contradictory findings. Project Motion aimed to rigorously examine MOBC in MI by improving on prior methods and using an enhanced conceptual framework that considered non-specific therapy factors and self-change mechanisms. To accomplish this, Project Motion was a small pilot study that disaggregated MI into its component parts and tested full MI compared to MI without its directive strategies. This comparison aimed to determine whether the directive elements of MI are critical or whether MI effects may be attributable solely to its Rogerian, non-directive components (therapist empathy, genuineness). Rogerian elements are commonly referred to as MI spirit [28]. Thus, the treatment conditions were labeled Full MI (FMI) and Spirit-Only MI (SOMI). In addition, it was tested whether hypothesized main effects of FMI were mediated via increases in commitment to reduce drinking early in treatment using state-of-the-art assessment methods, such as daily data collection using the Interactive Voice Recording (IVR).

Recruitment. General advertising online and in local media was used to recruit 89 participants seeking to reduce but not stop drinking. Advertisements emphasized a moderation approach and avoided labeling drinking as a problem. In response to advertisements, individuals called into the main study telephone number and were provided initial information about the project. Once the caller provided consent over the phone, he or she was then assessed for initial study eligibility. Eligible participants were then scheduled for a full in-person assessment with study staff.

Study eligibility. Participants were considered eligible for the study if they were: (1) between the ages of 18 and 65; (2) had an estimated average weekly consumption of greater than 15 or 24 standard drinks per week for women and men, respectively; and (3) had a primary alcohol use
Table 1: Baseline characteristics of study sample (N = 89)

<table>
<thead>
<tr>
<th>Variable</th>
<th>MI (N = 29)</th>
<th>SOMI (N = 30)</th>
<th>SC (N = 30)</th>
<th>Overall Sample (N = 89)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age (years) ± SD</td>
<td>40.8 ± 11.9</td>
<td>39.8 ± 11.8</td>
<td>37.4 ± 11.4</td>
<td>39.2 ± 11.7</td>
</tr>
<tr>
<td>Male</td>
<td>41.4</td>
<td>50.0</td>
<td>60.0</td>
<td>50.6</td>
</tr>
<tr>
<td>Race/Ethnicity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Hispanic, White/Caucasian</td>
<td>79.3</td>
<td>86.7</td>
<td>80.0</td>
<td>82.0</td>
</tr>
<tr>
<td>Hispanic/Latino, any race</td>
<td>6.8</td>
<td>10.0</td>
<td>10.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Other</td>
<td>13.7</td>
<td>3.3</td>
<td>10.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High school diploma, GED and under</td>
<td>3.4</td>
<td>6.7</td>
<td>6.6</td>
<td>4.5</td>
</tr>
<tr>
<td>Some college/Associate’s</td>
<td>27.6</td>
<td>23.4</td>
<td>16.7</td>
<td>22.5</td>
</tr>
<tr>
<td>Bachelor’s degree</td>
<td>31.0</td>
<td>36.7</td>
<td>43.3</td>
<td>37.1</td>
</tr>
<tr>
<td>Some graduate school or higher</td>
<td>37.9</td>
<td>33.3</td>
<td>33.3</td>
<td>34.8</td>
</tr>
<tr>
<td>Employment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>89.6</td>
<td>90.0</td>
<td>76.7</td>
<td>65.2</td>
</tr>
<tr>
<td>Unemployed/ Looking for work</td>
<td>6.9</td>
<td>3.3</td>
<td>13.3</td>
<td>7.9</td>
</tr>
<tr>
<td>Not in labor force/ not looking for work</td>
<td>3.4</td>
<td>6.7</td>
<td>10.0</td>
<td>6.7</td>
</tr>
<tr>
<td>Drinking severity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean sum of standard drinks per week (± SD)</td>
<td>33.9 ± 20.4</td>
<td>31.2 ± 19.0</td>
<td>29.2 ± 12.9</td>
<td>31.4 ± 17.6</td>
</tr>
<tr>
<td>Mean drinks per drinking day (± SD)</td>
<td>6.9 ± 3.0</td>
<td>5.7 ± 2.6</td>
<td>5.8 ± 2.8</td>
<td>5.9 ± 2.8</td>
</tr>
<tr>
<td>Short Inventory of Problems (± SD)</td>
<td>16.5 ± 8.9</td>
<td>15.0 ± 7.1</td>
<td>13.2 ± 5.2</td>
<td>14.9 ± 7.2</td>
</tr>
<tr>
<td>Alcohol Dependence Scale (± SD)</td>
<td>13.6 ± 4.6</td>
<td>11.8 ± 5.3</td>
<td>12.1 ± 5.3</td>
<td>12.5 ± 5.1</td>
</tr>
<tr>
<td>Number of alcohol dependence criteria met (± SD)</td>
<td>4.6 ± 1.5</td>
<td>3.6 ± 1.6</td>
<td>3.7 ± 1.4</td>
<td>4.0 ± 1.6</td>
</tr>
<tr>
<td>F(2,86) = 3.67, p &lt; 0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Any drug use</td>
<td>34.5</td>
<td>43.3</td>
<td>43.3</td>
<td>40.4</td>
</tr>
<tr>
<td>Beck’s Depression Inventory-II Score (± SD)</td>
<td>15.2 ± 9.8</td>
<td>12.2 ± 8.2</td>
<td>11.0 ± 7.2</td>
<td>12.8 ± 8.5</td>
</tr>
<tr>
<td>Ever received formal treatment for substance abuse</td>
<td>31.0</td>
<td>16.7</td>
<td>13.3</td>
<td>20.2</td>
</tr>
</tbody>
</table>
disorder (AUD). Participants were excluded from the study if they: (1) presented with significant substance use or a current substance use disorder (for any substance other than alcohol, marijuana, nicotine or caffeine), which was defined for our assessment purposes as greater than once weekly use in the past month; (2) presented with a serious psychiatric illness or substantial suicide or violence risk; (3) demonstrated clinically severe alcoholism, as evidenced by physical withdrawal symptoms or a history of serious withdrawal symptoms; (4) were legally mandated to complete a substance abuse treatment program; (5) reported social instability (e.g., homeless); (6) expressed a desire to achieve abstinence at baseline; or (7) expressed a desire or intent to obtain additional substance abuse treatment while in the study. See Table 1 for baseline characteristics of the final Project Motion sample.

**Fixed Assessments and Randomization to Condition.** Treatment was delivered in 4 sessions that lasted between 45 minutes to an hour long at weeks 1, 2, 4, and 8. Participants were blind to condition assignment, meaning they were not told which therapy they would be receiving. During their initial in-person assessment, participants provided informed consent and participated in a full evaluation with a research assistant (RA) and a mental health clinician who assessed for any high risk mental health disorders, such as major depression. At the end of this evaluation (week 0), participants were trained on the interactive daily questionnaire system (called interactive voice recording (IVR), described further below) and asked to come in one week later to attend the full baseline assessment (week 1). At week 1, participants were: (1) provided general feedback about their drinking and given normative comparisons according to NIAAA guidelines [31] by the RA, and (2) randomly assigned to one of three conditions: Motivational Interviewing (MI), Spirit only Motivational Interviewing (SOMI), or Self Monitoring only. All participants were followed for a total of 9 weeks and participated in 90 minute assessments at weeks 0, 1, 4 and 8. Participants in MI and SOMI were called an additional 4 weeks post-treatment (week 12) to collect drinking data via the Timeline Follow Back (TLFB, described further below). Follow up rates for assessments at weeks 1, 4, 8 and 12 were 100%, 96%, 92.1%, and 68% respectively.

**Daily Questionnaire/Interactive Voice Recording (IVR) Procedures.** All participants were asked to complete a daily telephone survey at the end of each day for the duration of the eight weeks (one week of baseline information and seven weeks of treatment) of the study, for a total of 56 possible days. At the end of their initial screening visit (a week prior to randomization), a research assistant (RA) provided each participant a 15 minute training session on how to use the Interactive Voice Recording (IVR) system, a system developed using TELESAGE SmartQ 5.2, a software package specifically designed for the administration of automated surveys [38]. Each day, participants responded to a series of questions about potential mediators of drinking behavior such as mood, commitment to not drinking heavily or not at all, and confidence to do so, in addition to the number and type of drinks they consumed in the last 24 hours. Once familiar with the system, the daily IVR session required approximately 5-minutes to complete. Each participant was provided a toll-free phone number and an anonymous participant identification number to ensure confidentiality. The IVR system could be accessed between 4:00 p.m. and 10:00 p.m. This time period was judged to be when participants most likely would be able to reflect on their alcohol use that occurred past the prior days assessment and before most individuals would be likely to consume large amounts of alcohol. This time window had the advantage of providing consistent report timing and facilitated compliance by creating routines for participants. If participants failed to call into the system by 8:00 p.m., an automated reminder call was made. Participants data were coded as missing if they were not able to complete a call. This daily questionnaire was considered a form of self monitoring ones drinking.
3 Interpretation of Data to Inform Modeling

While the wealth of information in this rich data set presents a unique opportunity to examine aspects of drinking behavior change longitudinally, it also presents challenges. In contrast to other areas, such as the physical sciences, in which the state variables representing key players in the processes are clear, no such information is known. In addition, the relevant timescales on which the variables are changing and therefore should be observed are unclear. It is possible that some of the measures, if not deemed to be important on the timescale of the current data set, may be driving other aspects of drinking behavior either on a shorter or longer timescale than that on which we choose to focus.

As we initially approached the task of developing dynamical systems models of these processes, we were prompted to think critically about the information content in the Project MOTION data. Individuals’ drinking behavior, and their response to treatment, is highly variable, and it is likely that one model describing all individuals is not possible. At any rate, to be so ambitious with first attempts at discerning key mechanisms is unreasonable and focusing on groups of individuals who respond in similar ways is a more feasible approach. With that in mind, we proceeded to organize the data in ways that would best inform modeling efforts, described in much greater detail in [9].

With the frequency of the IVR responses, it is natural to focus on those data for the development of a dynamic model. We also considered which of the fixed assessment battery provides novel or equivalent information to that in the IVR. While these data are not sufficiently informative on a longitudinal basis to be considered as main components of a model at this stage, it is possible that these data could’ve supplemented possible missing responses from the IVR, or the novel data could be information that should be included in future IVR-type questionnaires. The details of this exploration are given in [9], but ultimately, we did not use any of this information in the models presented here.

The timeline followback responses proved to be useful in that we were able to determine criteria upon which to divide the subjects into ‘responders’, ‘non-responders’, or neither. We defined responders as those individuals who demonstrated at least a 40% decrease in their drinking by their last treatment session or the week 8 assessment. Non-responders were identified as demonstrating no change in their drinking or increasing their drinking by the week 8 assessment. Thus, there are several individuals who would be classified neither as a responder nor as a non-responder. We thought it prudent to focus on individuals with the desired behavior-changing mechanisms in place, or at least those who exhibited the desired behavior changes, for initial modeling efforts. Therefore, we chose to focus on the eight subjects classified as ‘responders’. With these individuals’ data, we determined an initial grouping of IVR data questions (as it is not reasonable to consider each of the 42 questions as independent information, or variables) based on intuition, categories the investigators had in mind when constructing the IVR questionnaire, and previous hypotheses of interactions of key factors, described in more detail in [9], and also as the focus of the next section.

4 Static statistical methods

In the hopes of identifying useful data categories, or groups of similar IVR items, to be used as variables, we turned to common statistical techniques, which are commonly used in the analysis of such data sets among psychologists. The techniques employed were least-squares regression, principal component analysis, and factor analysis. We note that these test for static relationships, and we would not expect them to be very informative concerning processes that change in anything more sophisticated than linear with time.
4.1 Least-squares determined interpolation methods

Scatter plots, multivariate least squares regression, and \( r^2 \) statistics related to the regression models were the first tools we used to inform us of relationships between data and also between data categories. As explained in [20], given data \( X \in \mathbb{R}^{m \times n} \) that contains \( m \) observations of \( n \) variables. For the variables \( x_i \), where \( i \in [1, 2, ..., n] \), with expectation \( \mathbb{E}(x_i) \) and standard deviation \( \sigma_i \), \( 1 \leq i \leq n \), a regression model is used to minimize the (least squares) error

\[
\text{err}(f(x)) = \sum_{k=1}^{m} |y_k - f(x_k)|^2
\]

where \( y^{(k)} \) is a single variable at observation \( k \), and \( f(x^{(k)}) \) is a general model depending on variables \( x^{(k)} \). In this investigation we used the affine regression model

\[
f(x_i, x_j) = c_0 + c_1 x_i + c_2 x_j + c_3 x_i x_j,
\]

where the interaction term \( x_i x_j \) is considered as its own dimension. We also considered affine relationships between two data items \( x_i \) and \( y \) by setting \( c_2 = c_3 = 0 \). The coefficients of (2) are found by minimizing (1) using

\[
X \bar{y} = X^T X \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \end{bmatrix}^T,
\]

where \( X = \begin{bmatrix} \bar{x}_i & \bar{x}_j & \bar{x}_i \cdot \bar{x}_j \end{bmatrix} \); \( \bar{x}_i, \bar{x}_j \), and \( \bar{y} \) are column vectors of data objects. To judge the strengths of these relationships between a response variable \( y \) and explanatory variables \( \bar{x} \) we utilized coefficients of determination \( r^2 \). The coefficient of determination is

\[
r^2_{i,j} = 1 - \frac{\sum_{k=1}^{n} (y_k - f(x_{k,i}, x_{k,j}))^2}{\sum_{k=1}^{n} (\mathbb{E}(y) - y_k)^2}.
\]

Since we expected that different participants in Project MOTION possess different characteristics, we focused on calculating \( r^2 \) between each of the variables and categories per patient. We sought strongly (\( r^2 > 0.50 \)) affine relationships and weaker (\( 0.20 < r^2 < 0.50 \)) affine relationships.

4.1.1 Interpolation of IVR data

We focused our search for affine relationships on the IVR data on 11 participants who often completed the IVR survey and decreased their drinking over the course of Project MOTION. The \( r^2 \) values per patient often varied, and the \( r^2 \) values per pair of variables averaged across individuals indicated that across this sample of participants, only 3 relationships were strongly affine and only 21 were weakly affine. All of these relationships occurred between questions that are in the same category, affirming that the grouping of data items into variables used here may account for a sufficient amount of variation in the IVR data sets. The operations performed when we attempted to formulate models for three variables on these data sets were rank deficient and thus gave unreliable results.

We then considered the relationships between data categories as expressed by their composite scores created from the data. Examining one composite score versus drinks did not reveal any clear relationships – except for possible relationships between time and drinks (\( r^2 = 0.1845 \) for one participant representative of the cohort) and between guilt and drinks (\( r^2 = 0.1879 \) for that one participant). Transformations in the form of (2) showed that when the drinks versus composite score data is spread along time, some relationships can be represented relatively well by an affine function, but most relationships still are weak, indicating that in general these relationships are not represented well by affine transformations. Figure 1 shows a typical result of attempting to fit an affine regression model to three variables.
Figure 1: Stress versus drinks versus time with affine interpolation \( A(t) = c_0 + c_1 S(t) + c_2 t + c_3 tS(t) \) for one patient. The model fails to account for a great amount of variability in the data.

4.2 Principal component analysis

Principal component analysis (PCA) is a multivariate data analysis tool that may be used to transform a set of potentially correlated data objects into a smaller set of uncorrelated objects via rotations and scaling of the original data. This smaller set of objects maintains a percentage of the information of the original data. A short explanation of PCA and example of application of PCA to a small data set is available in [23]; a more in-depth, implementation-oriented explanation and an example using SAS on the results of a questionnaire are in [35], and a thorough explanation is in [21]. We used PCA to determine if the topical data categories appropriately relate to the dynamics of drinking behavior, or if not, what variables may better explain the behavior.

We performed PCA on the IVR response data from six ‘responder’ patients. After the principal components were determined for each data set, the linear combinations that produce the components per data set were compared to detect similarities between the sets of components formed for each patient.

4.2.1 Mathematical background of PCA

Consider a data set \( \mathbf{X} \in \mathbb{R}^{m \times n} \), where the notation is consistent with that in Section 4.1. Through PCA, we find a matrix \( \mathbf{V} \) that transforms the original data set \( \mathbf{X} \) into \( \mathbf{Y} \) by \( \mathbf{Y} = \mathbf{XV} \) so that \( \mathbf{Y} \) expresses all of the information originally in \( \mathbf{X} \). The columns of \( \mathbf{V} \) are called components, and each element of \( \mathbf{V} \) is known as a loading. Typically, the columns of \( \mathbf{V} \) are orthogonal, corresponding to an orthogonal rotation of the data axes so that the new variables represented by the columns of \( \mathbf{Y} \) are uncorrelated and arranged in order of decreasing variance.

The use of PCA requires a few assumptions about the data set \( \mathbf{X} \). The primary assumption is that linear relationships exist between the data objects, and the calculation of error estimates, confidence intervals, and variance require that the original data \( \mathbf{X} \) are normally distributed. Each variable in \( \mathbf{X} \) should be standardized so that it is has mean 0 and variance 1. PCA is performed using either the covariance or the product-moment correlation matrix of \( \mathbf{X} \). The use of the covariance matrix is appropriate when data objects are measured in comparable units and the variance between data objects has practical significance that would be altered by normalization. The use of the
correlation matrix is recommended when data items are measured in different units or when the variances between data items have no practical significance and requires that all columns have been standardized to a mean of zero and variance of one. The correlation matrix is used for PCA on the Project MOTION IVR data.

The elements \( c_{i,j} \) of correlation matrix \( C \) are found by

\[
c_{i,j} = \frac{1}{\sigma_i \sigma_j (n - 1)} \left[ (\bar{x}_i - E(\bar{x}_i))^T (\bar{x}_j - E(\bar{x}_j)) \right].
\]

This matrix \( C \) is symmetric and positive semi-definite by its construction. We computed the eigenvalue and eigenvector pairs \( \lambda_i, \vec{v}_i \) that satisfy \( C \vec{v}_i = \lambda_i \vec{v}_i \) using singular value decomposition (SVD) on the normalized data. Each \( \vec{v}_i \) contains the weights of the data items that will be used for the calculation of the \( i^{th} \) component, the eigenvalue \( \lambda_i \) represents the amount of variance described by the variable corresponding to the \( i^{th} \) component, and the sum \( \sum_{j=1}^{n} \lambda_j \) is the total variance.

As noted earlier, not all of the components are included in the final set of components: failure to exclude some of the components is equivalent to no reduction in dimensionality of the data set. Several rules have been developed to determine the number of components to include \( p \) and are discussed in [35] and [21]. We selected enough components to account for approximately 80% of the total variance and also examined scree plots of the variances \( \lambda_i \).

4.2.2 Numerical implementation of principal component analysis

Principal component analysis via orthogonal rotation is most commonly implemented by performing singular value decomposition (SVD) on the normalized data matrix

\[
A = \begin{pmatrix}
\frac{x_1 - E(x_1)}{\sigma_1}, & \frac{x_2 - E(x_2)}{\sigma_2}, & \cdots, & \frac{x_n - E(x_n)}{\sigma_n}
\end{pmatrix},
\]

which is related to \( C \) by \((n - 1)C = (A)^T A\). A thorough discussion of SVD may be found in chapter 5 of [25]. Performing SVD on \( A \) yields \( A = U \Sigma V^T \), where \( V \) contains the right eigenvectors of \( A^T A \) and \( \Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n) \) contains the square roots of the eigenvalues of \( A^T A \). Additionally, \( U \) and \( V \) are orthogonal matrices. We determine the eigenvalues and eigenvectors of \( C \) via \((n - 1)C^{(2)} = V \Sigma^2 V^T \), where \( \lambda_i = \sigma_i^2 / (n - 1) \).

In the implementation of PCA used to process the IVR data, we first fill in all missing values with the average of their data items and normalize the data so that each data item has mean zero and variance one. We then perform principal component analysis via orthogonal rotation on this matrix and determine an appropriate number of components \( p \) via the scree method.

4.2.3 Results of performing PCA on IVR data

Performing PCA on a IVR response data enabled us to search for any underlying themes in the data that were not captured by our data categories. The results of PCA in this application, however, did not reveal any interesting traits. Figure 2 displays a bar for each of the first 20 components of patient 6002’s data and represents the typical outcome when performing PCA on IVR data. Each color in a bar corresponds to the sums of loadings of all variables in a particular category of data. These categories are time, events last night, events today, mood, perceived stress, desire to drink, commitment to not drink, and evaluation of drinking as excessive (drinking guilt). Wider blocks of color correspond to a heavier weighting of those columns in creating that component. In this case, no variable strongly contributes to any component. The similar progression through colors indicates that every component is composed of a weighted sum of nearly all variables with weights
of nearly equal magnitude, so these components do not show any unique relationships between variables.

We may also consider the scree plots, like Figure 2, which shows the variances explained by the first 10 components of patient 6002. In this case and most other data sets, the first component explains less than 20% of the total variance, and the first ten components explained less than 70% of the variance. Approximately 20 components were required to account for 90% of the variance.

Figure 2: Left: Topical sum of the magnitudes of coefficients of PCA performed on data items for the first 20 components of subject 6002. From bottom to top are time, events last night, events today, mood, stress, desire, commitment, and guilt. Right: The individual (bar) and cumulative (line) variance explained by the first 10 components of PCA performed on the data for subject 6002.

After consolidating the data into the observable data categories formulated earlier, we again utilized PCA in order to detect any underlying mechanics that relate two groups together. PCA, however, also performed poorly on the observed data categories. The first principal component accounted for approximately 20% to 30% of the variance in the data, but the makeup of this or any component did not point to any trends other than a rough average of all topics (Figure 3). In all, approximately seven of the maximum twelve components were required to explain 80% of the variance in the data.

Performing PCA on the IVR data did not reveal any affine relationships that could be useful when formulating a dynamical system model. In addition to performing PCA on full data and data categories, we performed PCA on only emotion variables, on per-week data, and on weekly data per patient. In all cases, a high number of components were required to account for sufficient amounts of variance, and no unique relationships between variables manifested. Similar to the results of the affine regression models, performing PCA informs us that it is unlikely that the variables measured by the IVR are not related by affine transformations.

4.3 Static methods conclusions

The static methods used to search for affine relationships between variables failed to produce strong evidence for any linear patterns. While regression models were informative in that some existing relationships between variables, or observed data categories were revealed, the nature of these relationships were not made clear. Results from principal component analysis did not yield any unique trends in patient data, and factor analysis failed to result in any completed computations, indicating that the method is ill-suited to this data set.
We also performed factor analysis, a method similar to PCA and discussed by R. J. Rummel in [33], on the data. While factor analysis may be implemented using any one of a variety of methods, we employed the maximum likelihood method. The factor analysis resulted in a Heywood case, indicating that factor analysis is not an appropriate method to use on this data set. It is likely that the non-normal distribution of data and the small number of possible values for most observations were negatively affecting the estimations of covariances and loadings. Namely, the covariance of the assumed error in the measurements is estimated in an iterative process when using the maximum likelihood method, so if a value on the diagonal of this covariance matrix becomes less than or equal to 0, the maximum likelihood method is converging to an inappropriate result.

As static linear models do not appear to adequately explain the patterns shown in the IVR data, we propose the use of a possibly nonlinear dynamical system model which as we shall see below requires delays and cumulative or hysteretic functions. Additionally, our static linear analysis did not show any clear patterns between IVR questions. As noted when explaining the data collected during project MOTION, some of the data provides redundant or closely related information, and thus it is reasonable to base our model variables on the categorical groupings of IVR questions.

5 Initial mathematical model

For all patients, it is reasonable to allow the composite scores of the categorical groupings (and hence the variables) to fluctuate on a continuum as opposed to a discrete set of numbers, as a discrete set may not completely reflect how an individual is feeling. For example, a person is not simply happy or sad, there are many strengths at which happiness or sadness may be felt that do not just correspond to five digits. Discrete scales simplify the data that may be gathered from questionnaires and make this data more understandable to a person performing simple analyses; however, when developing a dynamic model, the use of a continuum of values is preferred, and, as can be argued, more reasonable. For convenience, and also because the variables of interest often change in a continuous manner, we will consider the model as a function of continuous time as opposed to discrete time.

We determined that a ‘triweekly’ timescale, seen in Figure 4, would be most suitable to observe the processes. Daily data appeared to have too many fluctuations, so that any trends (and hence, mechanisms or dynamics) were not clear, and data averaged by week appeared to not show enough

Figure 3: Coefficient magnitude of category principal components of subjects 6029 (left) and 6090 (right).
Figure 4: Daily (top), triweekly (middle) and weekly (bottom) drink counts for patient 6029. Red triangles indicate weekend days, blue dots indicate weekdays.

dynamics, or that the information was essentially ‘averaged out’. The triweekly timescale consists of 3 time points per week: weekend (Friday night through Sunday prior to IVR call), Sunday and Monday, and Tues through Thurs.
5.1 Patient 6029 model

![Graph showing weekly totals of drinks as reported by patient 6029 in the Timeline Followback (TLFB) assessment and the IVR system.]

Figure 5: Weekly totals of drinks as reported by patient 6029 in the Timeline Followback (TLFB) assessment and the IVR system.

We initially chose to focus on patient 6029 since based on the TLFB assessment, seen in Figure 5, it appeared that his drinking significantly changed during the observed time period in a seemingly systematic way. That is, there appeared to be some heavy drinking initially that the individual gradually reduced so that there was consistently and noticeably less drinking at the end of the observation period. This patient responded consistently to the IVR data and appeared to generally exhibit characteristics that the clinicians would identify as indicators of successful behavior change.

Causal relationships between potential variables were determined based on visual inspection of the data, and ‘categorical models’ based upon these relationships were constructed for a select four responding patients. The categorical model is a schematic representation of model variables and their relationships. In general, whether categories were included depended entirely on whether the category appeared to be directly related to drinking (represented as daily averages over the 2 or 3 day time period). Relationships were considered among the variables that were initially selected as being pertinent to the preliminary categorical model. We did not discuss possible relationships among variables that did not appear to be drinking related, and therefore were not considered in these categorical models. So, it could be that some of the variables are related to each other but that relationship is not represented in the categorical model unless each of them is related to drinking. Solid lines denote ‘strong’ relationships are represented and ‘weaker’ relationships are represented with dotted lines. Arrows indicated causality or direction. In Figure 6, all relationships appear bidirectional – drinks consumed appeared to influence one’s desire, and desire also influenced the drinks consumed.

The representation of the categorical model can be seen in Figure 6. The graphs from which we decided there are noticeable relationships are shown in Figure 7. All relationships for this individual seemed to be relatively strong when present.

The initial model, given by
Figure 6: Categorical model for patient 6029 based on hypothesized relationships in the IVR data. ‘Strong’ relationships are represented by solid lines, and ‘weak’ relationships are represented by dashed lines. All relationships are taken to be bidirectional.

\[
\frac{d}{dt}A(t) = -a_{12}\chi_{(G>0)} \left( \int_{-\tau_1}^{0} G(t+s)\kappa_1(s)ds \right)^2 + a_{13}\chi_{(D>0)}D(t) - a_{14}(Q(t) + \chi_{(Q>0)}Q^2(t)) - a_{15}L(t) \\
\frac{d}{dt}G(t) = a_{21} \left( \int_{-\tau_2}^{0} A(t+s)\kappa_2(s)ds - (1 + c_1\chi_{W(t)})A^*_G \right) \\
\frac{d}{dt}D(t) = -a_{34}Q(t) - a_{35}L(t) - a_{32} \left[ \exp \left( \frac{1}{A^*_D} \int_{-\tau_3}^{0} G(t+s)\kappa_3(s)ds \right) - G^*_{D2} \right] \\
\frac{d}{dt}Q(t) = -a_{43} \left( 1 + \chi_{W(t)}\chi_{(D>0)} \right) D(t) - a_{41} \left[ \exp \left( A^*_{Q1}\min(0, (A(t-\tau_4) - A^*_{Q1})) \right) - (1 - \chi_{W(t)})A^*_{Q2} \right] \\
\frac{d}{dt}L(t) = a_{52}G(t) - a_{53}D(t) - a_{51} \left[ \exp \left( \frac{1}{A^*_L} \min(A^*_{L1}, (A(t-\tau_5) - A^*_{L1})) \right) - A^*_{L2} \right].
\]

was derived from pre-existing knowledge of drinking behavior processes, and was suggested by patterns in the IVR data. It includes all possible mechanisms considered in these investigations and we did not think it likely that all terms would be included in the end. However, there are some key features that will likely be necessary to include in even the simplest mathematical representation of
individual-level behavior change in general and specifically concerning one’s alcohol intake. Namely, these are nonlinearities, delayed and/or cumulative effects, and time-dependent threshold behavior.

For illustrative purposes, consider the equation describing rate of change of guilt:

\[
\frac{d}{dt} G(t) = a_{21} \left( \int_{-\tau_2}^{0} A(t + s) \kappa_2(s) ds - (1 + c_1 \chi_W(t)) A_G^{\tau_2} \right).
\]

This equation is the only one that is not modified as the model solutions were compared to data (described in Section 5.2), and exhibits most of these characteristics listed above. It can be interpreted as the individual’s ‘guilt’, or more accurately his feeling that his drinking over the previous day was excessive, increases with the number of drinks consumed since \(\tau_2\), if they exceed the threshold \((1 + \chi_W(t)) A_G^{\tau_2}\). We note that since \(A(t)\) is the drinking rate, the number of drinks consumed from \(\tau_2\) days ago to the present time is represented by \(\int_{-\tau_2}^{0} A(t + s) ds\). The function \(\kappa_2(s)\) is used to effectively weight times at which drinking may have a more or less significant effect on his guilt. For example, if \(\kappa_2(s)\) increases with \(s\) going from \(-\tau_2\) to 0, then this would indicate that his drinking rate at the present time more strongly influences his guilt than in the past. This particular individual’s (and likely most individuals’) idea of ‘excessive drinking’ changes from the weekday to the weekend, and thus the function \(\chi_W(t)\), which is equal to one during the weekend and zero during the week, changes the individual’s level of ‘acceptable drinking’ or his threshold from \(A_G^{\tau_2}\) during the week to \((1 + c_1)A_G^{\tau_2}\) during the weekend.
5.2 6029 model modification

Attempting to examine how well solutions agree with data is nontrivial. Numerous parameters are not readily interpreted, thus complicating the estimation of feasible ranges for their values. While we may be able to anticipate the model solution of each individual term, we do not know the solution of the model when all terms are included simultaneously. Further, we do not expect that all terms will be included in a final model which we consider the best description for the system as justified by the current data set. Therefore, we took a reductionist approach. We began with the simplest system of drinking behavior that we anticipated capable of producing solutions with reasonably close behavior to that seen in the data. Once we have reproduced the data as well as possible with the simple model, we added additional components necessary to capture features in the data not explained by the current model. This approach corresponds to a general modeling philosophy of including as much complexity as needed to capture the key features of a given system, but no more.

The overall goal of the project is to understand drinking behavior, so the simplest model necessarily includes the equation for the drinking rate \( \frac{d}{dt}A(t) \), and a mechanism driving alcohol consumption and also suppressing it. To have one without the other would lead to strictly increasing or decreasing alcohol consumption, and from the IVR data, we know this is not the case. We focused on the desire \( D(t) \) and ‘guilt’ \( G(t) \) (which may be more accurately interpreted as a norm violation measure, but still referred to here in shorthand as guilt) variables. These were identified as two variables that appeared to have the strongest relationships with drinks in the IVR data. Therefore, the simple model initially considered is

\[
\begin{align*}
\frac{d}{dt}A(t) &= -a_{12} \chi_{G>G^*}(G(t) - G^*) + a_{13} D(t), \quad (8) \\
\frac{d}{dt}G(t) &= a_{21} \left[ \int_{-\infty}^{0} A(t + s) - (1 + c_1 \chi_{W(t)}) A^* \right], \quad (9) \\
\frac{d}{dt}D(t) &= -a_{32} \chi_{G>G^*}(G(t) - G^*) \quad (10)
\end{align*}
\]

Immediately upon inspection of the equations (8) - (10) there is an issue in that the only driving force in the alcohol equation is the desire to drink. But the equation governing desire, equation (10), results in a non-increasing time course for desire \( D(t) \). Thus, as written, desire will not provide a driving force for drinking as it will not increase beyond its initial value. Also in the original model, the only way for the individual’s desire to increase is through a negation of essentially controlling mechanisms. Therefore, it appeared that the model was missing a mechanism to drive the patient’s drinking, and upon further consultation with the data, a notable weekend/weekday pattern in both the desire and drinking data became apparent. That is, the individual’s desire to drink and his drinking increased going into the weekend and remained elevated during the weekend, and decreased as the weekend ends. Thus, we include the term \( c_2 h(\hat{t}) \) in the \( \frac{d}{dt}D(t) \) equation, where \( \hat{t} = t \mod 7 \) and \( h(\hat{t}) \) is given by

\[
h(\hat{t}) = \begin{cases} 
2(\hat{t} - 1.5) & 1.5 \leq \hat{t} < 2(\text{Fri a.m. thru Fri p.m.}) \\
-2(\hat{t} - 2.5) & 2 \leq \hat{t} < 2.5(\text{Fri p.m. thru Sat a.m.}) \\
-2(\hat{t} - 3.5) & 3.5 \leq \hat{t} < 4(\text{Sun a.m. thru Sun p.m.}) \\
2(\hat{t} - 4.5) & 4 \leq \hat{t} < 4.5(\text{Sun p.m. thru Mon a.m.}) \\
0 & \text{else}
\end{cases}
\]

A possible interpretation of this mechanism is discussed briefly in Section 5.4.
Setting $a_{32} = 0$ so that the desire equation is just $\frac{d}{dt} D(t) = c_2 h(\hat{t})$ allows us to see the effect of this weekend-dependent mechanism $h(t)$ on desire, shown in Figure 8. This form of the desire equation appeared to agree well with the desire data as seen in the bottom panel of Figure 9. Therefore, we have good reason to think that this subject’s desire is described well by our representation of the weekend/weekday mechanism alone, and we are therefore not motivated to include the term depending on guilt in this equation.

![Figure 8: Solution of $\frac{d}{dt} D(t) = c_2 h(\hat{t})$ and $c_2(\hat{t})$ with $c_2 = 4.25$ and $D(0) = -0.5$.](image)

Note, all discussions of *data fitting* in this document are not actually referring to a least-squares type of fitting, but rather the choosing of parameter values and manual adjustment to see the effects on the solution as compared with data, i.e., a type of simulation-based sensitivity analysis for model response with respect to parameter values. The parameter values must be somewhat close, and/or with feasible ranges determined to hope for any optimization routine to be able to minimize the difference between the model and data, thereby resulting in reasonable parameter estimates. Therefore, the manual ‘fitting’ discussed here is a necessary first step prior to estimating parameters via inverse problem methods.

Once the desire data was explained well, we examined the dependence of the individual’s drinking on the desire, which we had hypothesized as the driving mechanism. It became apparent that to have that term in the $\frac{d}{dt} A(t)$ equation depend on $D(t)$ was flawed since any positive value of $D(t)$ will result in an increase in drinking rate (i.e., drinking is accelerated). What is more accurate is that positive but possibly waning desire, for example on a Sunday, would mean that the individual is consuming alcohol albeit at a slower rate, and thus, their rate of drinking is positive $A(t) > 0$, but decreasing (say, as they go into the work week) $\frac{d}{dt} A(t) < 0$. This is directly proportional to the rate of change of desire, not the value of the desire variable. Intuitively, it does make sense that the rate of change of alcohol consumption should be more directly related to the rate of change of desire, and thus, the number of drinks should be correlated with the individual’s desire level. This is a subtle point, but it does require attention to maintain the fidelity of the processes represented in
the mathematical model. The simple model should more accurately be
\[
\frac{d}{dt}A(t) = -a_{12} \chi_{G>G^*}(G(t) - G^*) + a_{13} \frac{dD}{dt},
\]
\[
\frac{d}{dt}G(t) = a_{21} \left[ \int_{-1}^{0} A(t + s)ds - (1 + c_1 \chi_W(t))A^* \right].
\]
\[
\frac{d}{dt}D(t) = -a_{32} \chi_{G>G^*}(G(t) - G^*) + c_2 h(t).
\]

But this is redundant, and the system may be reduced by the substitution of the equation \( \frac{d}{dt}D(t) \) into the 2nd term of equation \( \frac{d}{dt}A(t) \), resulting in
\[
\frac{d}{dt}A(t) = -a_{12} \chi_{G>G^*}(G(t) - G^*) + a_{13} (-a_{32} \chi_{G>G^*}(G(t) - G^*) + c_2 h(t))
\]
\[
\frac{d}{dt}G(t) = a_{21} \left[ \int_{-1}^{0} A(t + s)ds - (1 + c_1 \chi_W(t))A^* \right].
\]

If we take \( \tilde{a}_{12} = a_{12} * a_{32} \) and \( \tilde{a}_{13} = a_{13} * c_2 \), the model is further simplified to
\[
\frac{d}{dt}A(t) = -\tilde{a}_{12} \chi_{G>G^*}(G(t) - G^*) + \tilde{a}_{13} h(t),
\]
\[
\frac{d}{dt}G(t) = a_{21} \left[ \int_{-1}^{0} A(t + s)ds - (1 + c_1 \chi_W(t))A^* \right].
\]

The drinking and guilt data are shown along with solutions to the model (14)-(15) with \( \tilde{a}_{12} = 0 \) in Figure 9, along with the desire data and the solution of \( \frac{d}{dt}D(t) = c_2 h(t) \). The parameters and initial conditions are given in Table 2. The agreement between the solution and data is good in the sense that drinking episodes (with the exception of two) are predicted, and suggests that the individual’s drinking may be primarily attributed to the weekend/weekday pattern. Further, it appears that his desire and drinking are closely linked, being that the same function \( h(t) \) can be used to reproduce both of the trajectories shown in the data.

In the interest of considering whether any improvement in agreement could be made by including the effect of guilt in the drinking equation \( \frac{d}{dt}A(t) \), various model solutions with values of \( \tilde{a}_{12} \neq 0 \) were plotted. Even for small nonzero values of \( \tilde{a}_{12} \), no improvement is evident and in fact, the agreement appears to worsen. For illustrative purposes, one such solution can be found in Figure 10, with \( \tilde{a}_{12} = 0.5 \), which is very small compared to \( \tilde{a}_{13} = 16 \). The effect of guilt on drinking only decreases the drinks slightly at their peak since guilt is otherwise below its threshold \( G^* \). This then decreases the effect of drinking on the guilt variable, as the individual’s norms are violated to a lesser extent. While we suspect this implies that the effect of guilt on drinking is relatively insignificant compared to the more dominant weekend-dependent effect seen in the desire data, we make use of an inverse problem technique in the next section to determine if it is reasonable to exclude this term in the model.
Figure 9: Agreement between the mathematical model and patient 6029’s triweekly IVR data.

Table 2: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{a}_{12}$</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{a}_{13}$</td>
<td>16</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$c_1$</td>
<td>8.1</td>
</tr>
<tr>
<td>$A^*$</td>
<td>1</td>
</tr>
<tr>
<td>$A(0)$</td>
<td>1</td>
</tr>
<tr>
<td>$c_2$</td>
<td>4.25</td>
</tr>
<tr>
<td>$G(0)$</td>
<td>0</td>
</tr>
<tr>
<td>$D(0)$</td>
<td>-0.5</td>
</tr>
</tbody>
</table>
Figure 10: Agreement between the mathematical model and patient 6029’s triweekly IVR data with $\tilde{a}_{12} > 0$ small.
Table 3: Some values for the $\chi^2(1)$ distribution

<table>
<thead>
<tr>
<th>$\chi^2(1)$</th>
<th>confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.32</td>
<td>75%</td>
</tr>
<tr>
<td>2.71</td>
<td>90%</td>
</tr>
<tr>
<td>3.84</td>
<td>95%</td>
</tr>
<tr>
<td>6.63</td>
<td>99%</td>
</tr>
<tr>
<td>10.83</td>
<td>99.9%</td>
</tr>
</tbody>
</table>

5.3 Model Comparison Statistic

The model comparison statistic used here (and described in much greater detail in [4]) can be used to determine if a more complicated version of a simpler model gives a statistically significant improvement in the agreement between model solution and data. That is, it is appropriate when comparing ‘nested’ models, or when one model can be written as a special case of the other. As such, it is well suited to help determine whether specific terms in models can reasonably be simplified, or even neglected altogether. We employ it here to determine whether the model

$$
\frac{d}{dt}A(t) = -a_{12}\chi_{G>G}(G(t) - G^*) + a_{13}h(t),
$$

$$
\frac{d}{dt}G(t) = a_{21}\left[\int_{1}^{0} A(t + s)ds - (1 + c_{1}\chi W(t))A^*\right].
$$

(where the tilde’s above parameters $a_{12}$ and $a_{13}$ are neglected here and throughout the rest of the paper) with $a_{12} \geq 0$ provides a statistically significant improvement, if any, to the model with $a_{12} = 0$ when comparing model solutions with data.

The statistic involves the residual sum of squares ($RSS$), which is a measure of the distance between the model solution and data and is given by

$$RSS = \sum_{i=1}^{n} \left| f^{(1)}(t_i; \theta) - y^{(1)}_i \right|^2 + \left| f^{(2)}(t_i; \theta) - y^{(2)}_i \right|^2,$$

where model parameters are represented by $\theta$, $\{y^{(1)}_i\}_{i=1}^{n_1}$ is the drinking data, $\{f^{(1)}(t_i; \theta)\}_{i=1}^{n_1} = \{\int_{1}^{0} A(t_i + s; \theta)ds\}_{i=1}^{n_1}$ computed with parameters $\theta$, $\{y^{(2)}_i\}_{i=1}^{n_2}$ is the guilt data, $\{f^{(2)}(t_i; \theta)\}_{i=1}^{n_2} = \{G(t_i; \theta)\}_{i=1}^{n_2}$ and $n_1 = n_2 = 23$ for the data on the triweekly timescale. The corresponding model values are also converted to the triweekly timescale in the equivalent way (averaging over 2 or 3 days as described in the end of Section 5).

We estimate model parameters $\theta$ by minimizing the objective functional

$$J(\theta) = \sum_{i=1}^{n_1} \left| f^{(1)}(t_i; \theta) - y^{(1)}_i \right|^2 + \sum_{i=1}^{n_2} \left| G(t_i) - y^{(2)}_i \right|^2,$$

over a feasible parameter space $\Theta$. In comparison, estimating parameters with the constraint $a_{12} = 0$ is done by minimizing the same functional over a restricted parameter space $\Theta^H = \{\theta \in \Theta | a_{12} = 0\}$ which is a subspace of the original space ($\Theta^H \subset \Theta$). This is a reduction in the degrees of freedom by one, and in general, the resulting residual sum of squares when allowing $a_{12}$ to vary (minimizing over $\Theta$) will be at least as small or smaller than when minimizing over $\Theta^H$, or fixing $a_{12} = 0$. The
test statistic can be used to determine whether this improvement, if any, is statistically significant and is defined by

\[ U = \frac{n(J(\theta^H) - J(\theta))}{J(\theta)} \]  

(17)

where \( n = n_1 + n_2 \).

We performed this test in two ways: comparing residuals from estimating model parameters \( \theta = (a_{12}, a_{21})^T \) with estimating \( \theta^H = (0, a_{21})^T \) (where \( a_{12} = 0 \) and \( a_{21} \) is allowed to vary), and \( \theta = (a_{12}, a_{13})^T \) compared with \( \theta^H = (0, a_{13})^T \). In the first case, the feasible parameter space is \( \Theta = \{[0, 10] \times [0, 2]\} \), and in the second case \( \Theta = \{[0, 10] \times [8, 20]\} \). The statistic when comparing the residuals for \( \theta = (a_{12}, a_{21})^T \) and \( \theta^H = (0, a_{21})^T \) is

\[ U = \frac{46(114.28 - 113.10)}{113.10} \approx 0.47993. \]

This statistic may then be compared to a \( \chi^2 \) distribution with one degree of freedom (select values of which can be seen in Table 3) to determine whether we reject the null hypothesis. For this example, the null hypothesis is that the restricted parameter space (with \( a_{12} = 0 \)) is the appropriate one. (We note this procedure does not allow us to accept the null hypothesis, but only to either reject it or not. A much more precise and thorough discussion of this test can be found in [4, 6, 7, 8].)

Therefore, we conclude that allowing \( a_{12} > 0 \) does not result in a significant improvement.

When beginning the minimization to estimate parameters \( \theta = (a_{12}, a_{13})^T \) we initialized the procedure with \( a_{12} = 0.1 \). However, the parameter value for \( a_{12} \) that minimized the objective functional was \( a_{12} = 0 \), and thus the residuals (RSS) were the same. Therefore, there was no need to compute the statistic and we can directly interpret that there is no improvement in the fit. Similarly, when estimating all three parameters \( \theta = (a_{12}, a_{13}, a_{21})^T \), as compared with estimating \( \theta^H = (0, a_{13}, a_{21})^T \), the difference in residuals (RSS) is approximately \( 10^{-6} \), which results in a model comparison statistic value (also approximately \( 10^{-6} \)) that indicates a very insignificant improvement to the fit. Therefore, we cannot reject the null hypothesis with any reasonable level of confidence. Thus, it is reasonable to take \( a_{12} = 0 \), which amounts to excluding the term \( -a_{12}\chi_{G>G'}(G(t) - G^*) \) in the \( \frac{d}{dt}A(t) \) equation for the final model of patient 6029.
5.4 Final 6029 model

The model that best describes the dynamics in the patient 6029’s data is

\[
\frac{d}{dt} A(t) = a_{13} h(t),
\]

\[
\frac{d}{dt} G(t) = a_{21} \left[ \int_{-1}^{0} A(t + s) ds - (1 + c_1 \chi W(t)) A^* \right].
\]

While there is always room for improvement in the development of a model, at this stage there is likely not much to be gained from the inclusion of additional mechanisms. We note that the modeled processes are not as inherently predictable as those in biology, physics, or engineering. Therefore, our goals and standards for agreement between model solutions and data should be adjusted accordingly. We deem the ‘fit’ seen in Figure 9 to be relatively good since the episodes of drinking are, with the exception of two, accurately predicted by the model. We surmise that it is not necessarily a reasonable goal to predict the number of drinks during the episodes, but rather, whether or not they occur. In future modeling efforts, it may be that the episodes not predicted by the model are especially instructive, as they may be indicative of mechanisms missing from the model.

This individual reduced his drinking substantially at the beginning of the study and not during the IVR observation period. Therefore, it is difficult to speculate on the associated mechanisms accompanying his behavior change. However, the characterization of his drinking after reduction is instructive. The individual appeared to not drink much, and also exhibited a corresponding low desire to drink during the week, but his desire and drinking increased substantially on the weekends. Incorporating the \( h(t) \) function accounted for this and allowed for considerable agreement between model solutions and data. One possible explanation and therefore justification of including this term (weekday/weekend) relates to the perceived availability, craving, and consumption literature. Field and Cox ([14]) in their review of this literature note that intensity of craving following cue exposure has been shown to vary depending on the participants’ belief that they would be able to consume the substance. For example, numerous experiments have shown that smokers who are told the opportunity to smoke is imminent exhibit a higher subjective craving than when they are told they will not be able to smoke for several hours. Perceived availability has also been used to explain differences in responses to cues among substance users continuing to use substances, and those seeking to remain abstinent [41]. We know from the clinical records that 6029 set a goal of reduced drinking on the weekday, but would allow himself to drink more heavily on the weekend. It may be that by clearly defining this limit, 6029 changed the perceived availability of heavy drinking and that his desire reflected when he felt he could drink. We can speculate from his clinical record and ability to reduce drinking that 6029 possessed strong internal control and perhaps this strong internal control was manifest in the ability to set limits such there was a corresponding reduction in desire. By contrast, someone with weaker internal control might set a limit, but still experience strong desire.

One puzzling question is why commitment and self-efficacy, variables hypothesized to be critical to control, do not appear in the equation. It may be that because 6029 expected to drink more on the weekend, he did not rate his commitment or self-efficacy lower during these periods. Alternatively, it may be that these variables are important determinants of change in the decision to reduce drinking, but that once a plan has been set in motion, they recede as operative variables, thus their fluctuation is not determinant to drink, unless the plan fails. We return to a fuller discussion of these issues in the discussion section.

It is striking that this individual’s drinking is described well by the weekend-dependent driving
mechanism, and also that his desire mirrors the same behavior. This close link may be an indication that his internal control is very good. That the other measured control variables (such as the confidence and commitment to limit drinking, or the commitment to abstain completely from drinking) do not appear to capture this information may suggest that these questions are not good indicators of internal control for this patient. Or it may be that there are other issues with the data collection process that do not allow for those relationships to surface in the data. At any rate, these efforts provide motivation to look at the measures of control further.

Figure 11: Updated categorical model for patient 6029

The first categorical model (Figure 6), which was provided as a schematic of hypothesized possible relationships between model variables, can now be updated to reflect the model (18) - (19), and is shown in Figure 11. While we did not use inverse problem methods to go from the more complex model to the resulting simple one, we did iterate between the model, comparisons between solutions and data, interpretation of data, and using current knowledge and plausible hypotheses to inform the model. Indeed, each arrow in the categorical model, indicative of terms in the mathematical model, represents a hypothesis about the underlying process. The decision to keep, discard, or refine the term is in a sense hypothesis testing, although it is not done in the usual formal way here. Typical statistical methods addressing hypothesis testing would be limited in their ability to reveal the dynamic relationships we focus on here as this type of information is lost as data is aggregated across individuals or responses (means, standard deviations, etc.). Mathematical modeling provides a way for us to precisely formulate hypotheses concerning dynamic variables. If we are able to make use of inverse problem methods and estimate model parameters, then we can use statistical methods in conjunction to test whether model modifications provide a statistically significant difference in the fit to data. These methods involve the calculation of a statistic based upon the residuals (the sum of the differences between the model solution and data point) from two models that have been fit to the same data set. Such statistically based model comparison techniques are discussed in Ch. 3 of [10].

It is encouraging to be able to arrive at such a relatively simple explanation of this patient's behavior, but this individual is atypical in a crucial way. Commonly amongst responding subjects in Project Motion, the individual experiences some pivotal or transformative period of change after which their alcohol consumption pattern is significantly changed. While this patient's drinking pattern is still not the picture of perfection (he continues to violate his norms on weekends), it does not change substantially during the time period of the IVR. It would likely be more instructive to study an individual who experiences this pivotal change during the IVR period so that we would have observations before, during, and after it to better characterize this process of change.

While we should not expect a simple model to describe a typical problem drinker, (indeed the close connection between their desire and actual drinking behavior is likely not present), our ability to methodically do so in this case demonstrates mathematical modeling as a useful tool in
understanding these data. Notably, it appears to provide a significant advantage over the use of static methods in which the identification of important factors (without attempting to determine precise relationships between them) relevant to the patient’s drinking behavior proved difficult.

6 Patient 6009 model

Patient 6009 was selected as the next model subject because his drinking data displayed a clear and consistent downward trend. Therefore, we hoped that information on any pivotal experience he may have had preceding and during his drinking reduction may be observed in the data. Additionally, this individual was homeless and unemployed when he enrolled in the study and was able to secure stable housing during the IVR, in which he was not allowed to drink. Therefore, we expected that the supporting mechanisms of his behavior change would be different than those seen in patient 6029. Namely, patient 6029 appeared to exhibit strong internal control, whereas it is possible that patient 6009 was able to reduce his drinking as a result of external factors.

We considered that there may be other mechanisms not considered in the initial categorical model or in the IVR data categories, such as the weekend-dependent desire mechanism that ended up key to understanding the patient 6029’s desire and drinking data. Of the data categories included in the categorical model, we looked to single question responses for possible relationships with drinks, as the variables in the resulting model for patient 6029 are from single questions, discussed in more detail in Section 8. In the end, the simple model of patient 6029 agrees with the dominating characteristics documented in the supporting notes of the clinical team. As a result, we begin by using the clinical summary of the patient, along with those selected to be in the initial categorical model (Figure 12) to select for items in the IVR that may be most relevant to his drinking. We then looked for whether any dynamic relationships appeared to exist between these variables and his drinking data, and thereby selected for inclusion in the model. This selection is described in Section 6.3. We note that this modification of our approach - use of single items and clinical summary data - is characteristic of the iterative nature of modeling. Each modeling exercise reveals new information which is then used to inform the next model. The construction of the model from plausible mechanisms and observed patterns in the IVR data is discussed in Section 7.

6.1 Patient Summary

Reviewing the clinical staff’s summary, we delineate here information we thought important in constructing a model for patient 6009. He reported his drinking as being affected by feelings of depression and it was suggested that drinking was largely a way to fill unstructured time as a result of being jobless (and homeless). These two factors are, of course, not entirely independent. These observations suggest that this individual has a relatively constant desire to drink and therefore we would not expect a weekend/weekday pattern as with Patient 6029. This desire likely stems from and is proportional to his feelings of depression and/or his frustration with his living and employment situation. Thus, we were motivated to examine the mood questions that may show evidence of depression or helplessness, and also the questions inquiring about one’s ability to handle problems and feelings of control they have over their life.

His situation (and thus, his drinking behavior), seemed to reach a pivotal point as he made productive changes in his life, such as finding employment and stable housing, the latter of which we know he accomplished somewhere between 10 and 20 days after he began participating in the IVR. Thus, we would expect that this individual has changed his drinking behavior by having better control over his somewhat constant desire. We examine the commitment and confidence responses
and also a possible relationship between these questions and the response to the question: “Did you meet a goal or accomplish a task that left you with a sense of accomplishment?”

### 6.2 Previous categorical model

The original categorical model is shown in Figure 12. We selected single questions from the IVR as potential variables as suggested by this categorical model, and also with what we know about the patient from his clinical summary.

![Figure 12: Categorical model for patient 6009 from initial efforts, discussed further in [9].](image)

‘Pleasant Events’ is the average of responses to similar questions. We anticipated the question asking about the accomplishment associated with meeting a goal would be reflective of the individual making productive changes in his life. The other questions inquire about pleasant interactions with friends, family, etc. and while they may be secondarily affected (i.e., they may be improved as a result of his improved living situation), the most direct effect is likely the feeling of accomplishment, and therefore we deemed this response the most likely candidate for a model variable.

Commitment to quit could have possibly reflected the individual’s control and commitment to behavior change as a dominant mechanism. Another measured response that is even more likely to be closely associated with drinking behavior is the individual’s commitment to limit their drinking. However, this information was averaged with their confidence in their ability to limit their drinking after it was noted that these two pieces of information are similar. Therefore, all three responses are examined separately in Section 6.3.

The questions dealing with feelings that may be indicative of depression (and part of the category grouping ‘Mood’) are those inquiring about the individual being sad, lonely, nervous, and bored. Each of these responses were considered as possibly reflecting the effect of depression on drinking.
6.3 Selection of variables

Here we discuss the observed pattern of drinking and hypothesize mechanisms that could be driving the dynamics based on patterns in the IVR data from the questions we’ve identified as most likely to play a major role in his drinking. As such, this discussion is largely to illustrate the effects of the other variables on drinking, thus the relationships reflected in the $\frac{dA}{dt}$ equation. Data responses are plotted along with drinks in the same panel but due to the different scales of drinking, which reaches values slightly less than 15 and most responses, which do not exceed values of 4, are plotted using both vertical axes. The left vertical axis always corresponds to the drink values, and the right axis corresponds to the IVR question response value.

We consider each of the selected IVR questions based on the clinical summary and initial categorical model along with drinking and look for possible dynamic relationships, or if it appears that a variable may track certain aspects of his drinking. This section is a discussion of how we determined that the best candidates are guilt $G(t)$, loneliness $L(t)$, confidence in his ability to resist drinking heavily $C_1(t)$, commitment to resist drinking heavily $C_2(t)$, and commitment to completely abstain from drinking $C_3(t)$. We considered also desire, as it reflected the most important driving mechanism for patient 6029 (even though it was not needed for the final model), and we discuss briefly why that variable, along with his responses of being sad, nervous, or bored, and responses to the stressful events questions are neglected for the development of a model at these initial stages.

![Figure 13](image.png)

Figure 13: Weekday and weekend drinks plotted on the triweekly timescale.

Not surprisingly as seen in Figure 13, patient 6009 did not exhibit a clear weekday versus weekend pattern. This lack of pattern was first suggested in his case summary in which it was noted that he had a lot of unstructured time (that he filled by drinking) due to being unemployed. The timescale was kept consistent with the triweekly timescale of patient 6029, as every few days is still the desired resolution and it is likely that weekends are significant events for most individual’s drinking behavior.

Guilt $G(t)$, or the feeling that his drinking since that time yesterday was excessive, is shown
in the top panel along with drinking in Figure 14. The guilt response nearly mirrors the drinking behavior, which is similar to the relationship seen in patient 6029’s responses. The difference, however, is that the time-dependence of his ‘acceptable threshold’ $A^*(t)$ appears to be independent of the weekend, but rather changes somewhere between $t = 11$ and $t = 14$, and could perhaps be indicative of a transforming experience that may have initiated his drink reduction. Regardless of the reason, a change in his attitude toward acceptable drinking levels appears to have occurred. Being that the relationship is similar to that seen with patient 6029 and also that guilt is directly a reflection on, or reaction to his drinking, we initially considered the equation $\frac{dG}{dt}$ as having the same form as that in the model for patient 6029. With his data, a uni-directional relationship fit the data best, with alcohol affecting the rate of guilt, but not vice versa. The coefficient of a term dependent on $G(t)$ was determined to be small or zero in the $\frac{dA}{dt}$ equation for patient 6029, suggesting that such an effect was not dominant in the dynamics. It is possible that guilt may affect other variables, which then affect drinking, but from comparisons not shown here between the IVR data of this and the other supporting variables (loneliness $L(t)$, meeting a goal $M(t)$, confidence in his ability to resist drinking heavily $C_1(t)$, commitment to resist drinking heavily $C_2(t)$, and commitment to completely abstain from drinking $C_3(t)$), no striking relationships between their longitudinal fluctuations were evident.

The patient’s desire $D(t)$, which is a ranking of the extent to which he agrees with the statement, “The idea of drinking is appealing”, is shown in the bottom panel of Figure 14. While there are some
similarities in dynamics, and there could be a relationship between this variable and drinking, it is not sufficiently distinct to be a good candidate for a preliminary model. That is, there are no clear relationships between the fluctuations in these two variables over time. While it is counterintuitive to think that the desire to drink is not related to drinking, without a clear relationship between the two variables from data, we are hesitant to include it in such preliminary modeling efforts. Later efforts should certainly explore why there is not a direct link between these two variables.

Figure 15: Drinks (left axis) and mood variables (right axis) (sad, nervous, lonely, and bored); Participants are asked to what extent the mood adjective describes them on a scale of 0 to 4: 0 - not at all, 1 - slightly, 2 - moderately, 3 - very much, 4 - extremely.

The case summary for patient 6009 stated that mood may have been a predominant influence in his drinking behavior since he reported being depressed in some of his sessions. Therefore, we examined the responses to questions in the previously averaged variable ‘Mood’, that may indicate depression. The patient’s response to the extent to which they feel ‘sad’, ‘nervous’, ‘lonely’, and ‘bored’ are shown in Figure 15. In all three of the ‘sad’, ‘nervous’, and ‘bored’ responses the responses were generally low ($\leq 2$ which is only a ‘moderately’ response). In addition, the
fluctuations in the responses did not seem to generally reflect that seen in drinking behavior. For example, the one significant report of the patient being sad actually corresponds to a day of abstaining from drinking, and other increases in sadness, just a few time points later, correspond to an increase in drinking.

In contrast, loneliness appears to have a strong relationship with drinking behavior. A general pattern throughout the observed time period is that drinking and loneliness increase simultaneously. Another striking feature of these dynamics is that loneliness and drinking also peak and then decrease twice (around \( t \approx 11 \) and \( t \approx 35 \)). One interpretation of this is that 6009's heavy drinking increases his sense of loneliness and isolation and that the intensity of these negative feelings may lead him to reduce his drinking when such feelings reach a threshold level as appears to occur on around day 11 and 35. We note that this explanation differs somewhat from the more classic affect regulation model that postulates that drinking increases to cope with negative affect. Instead, we postulate that during a change episode, individuals experience certain peaks in negative affect or negative experiences around drinking that lead them to reduce drinking. One might refer to these as pivotal moments of "insight" where drinking is connected in the moment to something unbearably negative which then provides a catalyst to change.

Both the confidence and commitment (Figure 16) to limit drinking reflect a vacillating response, possibly indicative of good intentions that lack resolve, and perhaps external events that are not captured by the IVR questionnaire. But notably, an increase in both responses occurred around the individual's first pivotal point \( \hat{t}_1 \), and is generally higher, although oscillating until around time \( t \approx 45 \) after which both responses are relatively high. These responses being both high simultaneously correspond to drinking being controlled after \( t \approx 45 \) which may be the second turning point \( \hat{t}_2 \), in which patient 6009 is able to make significant improvements to his life or in which he is able to effectively change his behavior.

Additionally, the commitment to abstain is low in general, reflecting that patient 6009 does not wish to quit drinking, but rather to limit it to a more acceptable level. What is striking is high values of this variable do indeed correspond to the rare time points at which he does not drink at all. This variable is therefore indeed correlated with this patient abstaining from drinking and can explain the occasions in which he completely abstains from drinking which cannot be explained by other variables.

The responses to the question "Did you meet a goal or complete a task that left you with a sense of accomplishment?" seem to predict drinking behavior, as it appears that the highest response immediately precedes the individuals first turning point at \( t = \hat{t}_1 \), seen in Figure 17. Also, whenever the individual reports no accomplishment \( M(t) = 0 \), it precedes an increase in drinking. While there does appear to be a relationship between the longitudinal fluctuations in this variable and those in the individual's drinking, we also consider the scoring of this response and that a higher number does not signify a more intense feeling of accomplishment. A higher number may be indicative of more goals being accomplished, but is certainly not proportional to exactly how many. That is, if he meets six goals in one day and the next day he meets one goal in a day (and no change in his goals met in the evening), then his response is unchanged. In addition, the fluctuations in the alcohol data that we would anticipate this variable explaining are also closely linked to the trends in the loneliness \( L(t) \) variable. For these two reasons, this variable is left out of an initial model of patient 6009's dynamics.

In general, the responses to the stress questions (Figure 18) were relatively low, and no clear dynamic pattern is evident. Therefore, it is reasonable to neglect these variables in developing an initial model of 6009's drinking behavior.
Figure 16: Drinks (left axis) and the confidence and commitment responses (right axis) and all responses are on a scale of 0 to 4: 0 - not at all, 1 - somewhat, 2 - moderately, 3 - very, 4 - totally; Confidence to limit represents the patient’s response to how confident the individual is that he/she can resist drinking heavily; Commitment to limit represents the response to the inquiry of how committed he/she is to not drink heavily; Commitment to abstain represents the response to how committed the subject is not to drink at all.
Figure 17: Drinks (left axis) and Accomplishment (right axis); Responses to the question “Did you meet a goal or complete a task that left you with a sense of accomplishment?” could possibly be: 0 - no, 1 - yes, 2 - yes, but last night only, 3 - yes, both last night and today. This variable has been rescaled to: 0 - no, 1 - yes, either today or last night only, 2 - yes, both last night and today.
Figure 18: Drinks (left axis) and Stress questions (right axis) clockwise from top left: the extent to which the subject feels that they are unable to control the important things in their life, confidence about their ability to handle their personal problems, that difficulties are piling up so high that they cannot overcome them, and that things are going their way. Possible responses are 0 - not at all, 1 - slightly, 2 - moderately, 3 - very much, 4 - extremely.
We determined that guilt $G(t)$, loneliness $L(t)$, confidence in his ability to resist drinking heavily $C_1(t)$, commitment to resist drinking heavily $C_2(t)$, and commitment to completely resist drinking $C_3(t)$ are the best candidates for state variables for a model that would describe patient 6009’s drinking behavior. With the exception of guilt $G(t)$, these variables were selected based on the effects that they appear to have on drinking. That is, there were notable features in the evolution of these variables in time that seemed to correlate with a feature in the drinks consumed over time, along with a plausible explanation for such an effect.

For a complete dynamical system, we would like to also be able to derive equations for all state variables, and therefore we would need to be able to attribute changes in these variables to changes in other state variables, or to other time-dependent functions as we did with the $h(t)$ function for patient 6029. The other state variables may depend on other variables not in the current model (and therefore, not directly related to drinking), but may also be influenced by external events and/or internal processes not captured in the IVR data. Regardless, it does not appear that the fluctuations in these variables are related in any clear way to the other variables in the model as can be seen when comparing the data for the supporting variables directly. Therefore, we deemed
it not likely that we would be able to construct model equations for these variables depending on other model variables. However, we constructed functions for these variables that captured key aspects of their dynamics, similar to the function \( h(t) \) that reflected the observed weekend-weekday pattern for patient 6029. That is, we constructed ‘best fit’ lines through these data (piecewise linear functions) and used these functions as inputs into the equations for drinking rate \( A(t) \) and guilt \( G(t) \) that give a reasonable approximation to the observed data thereby suggest that these data are observations of dynamic processes. Further, the use of these dynamic data to reproduce dynamics in drinking and guilt data provide encouraging support for the use of mathematical models to provide meaningful explanations for these observations.

An updated categorical model, or schematic of the hypothesized relationships between variables is shown in Figure 19. Supporting variables, or variables that are used as inputs only are represented by boxes with dotted borders, and causal relationships are again represented by arrows. The appearance of the lack of relationships between support variables in this schematic is not to say that they do not exist, but that we are unable to determine them with our knowledge at this time and from the data set on hand.

We construct an initial model by examining the individual’s IVR response data, considering the previous experience modeling patient 6029, and constructing quantitative relationships that represent mechanisms consistent with our knowledge of processes underlying drinking behavior. We represent the mechanisms governing the state variables \( A(t) \) and \( G(t) \) by

\[
\frac{d}{dt} A(t) = c_1 + a_{13} (L^*(t) - L(t)) - a_{17} \chi_{C_2 > C_3} C_3(t) \\
- a_{15} \chi_{C_1 > C_1^*} \chi_{C_2 > C_2^*} (C_1(t) - C_1^*) (C_2(t) - C_2^*) , \quad (20)
\]

\[
\frac{d}{dt} G(t) = a_{21} \left( \int_{-\tau}^{0} A(t + s) ds - A^*(t) \right) , \quad (21)
\]

7.1 Drinking \( \frac{dA}{dt} \) equation

The equation, \( \frac{dA}{dt} \), which is the change in the individual’s rate of drinking with respect to time, is our primary focus since understanding the individual’s drinking behavior is our goal. The first constant term, \( c_1 \) represents the individuals’ relatively constant desire to drink, so is an ever-present source driving his drinking up. This is an unknown quantity, but we can determine a range of values that this parameter could possible take on (a necessary first step in doing computations and also for parameter estimation, if we were to pursue it). The only other variable that causes an increase in drinking is the feeling of loneliness \( L(t) \) (likely relating to depression). The drinking appears to increase proportional to loneliness, until it reaches a certain threshold, which may be interpreted as perhaps an unbearable level. The first of these switches occurs around \( t = 11 \), and twice more at \( t = 32 \) and \( t = 49 \). It appears that the threshold level of loneliness does not change until \( t = 32 \), at which point it decreases substantially. So \( L^*(t) \) is represented by the piecewise constant function

\[
L^*(t) = \begin{cases} 
L_1^* & 0 < t \leq 32 \\
L_2^* & t > 32 
\end{cases}
\]

and the constants \( L_1^* \) and \( L_2^* \) are apparent from the IVR data, and are therefore known values. Loneliness less than the unbearable threshold \( L < L^*(t) \) causes an increase in drinking, and the drinking decreases once this threshold is surpassed, thus the term is \( a_{13} (L^*(t) - L) \).
A further controlling mechanism for patient 6009’s drinking is reflected in his confidence in his ability to resist drinking heavily \( C_1(t) \), and his commitment to resist drinking heavily \( C_2(t) \). These variables induce a decrease in drinking only when both are above a threshold level, \( C_1^* \) and \( C_2^* \), which should be at least a response of ‘moderately’ or the numerical value 2. This effect is represented by the term \( -a_{15} \chi_{\{C_1^* \}}(C_1 - C_1^*)(C_2 - C_2^*) \). The product of the characteristic functions \( \chi_{\{C_1>C_1^*\}} \) and \( \chi_{\{C_2>C_2^*\}} \) will only ‘turn the term on’, i.e., be equal to 1 when both functions are 1. The effect when both commitment and confident to resist heavily drinking appears to be quite strong and is likely well represented by their product. It is possible that this effect may better be modeled by a saturating effect, in which increases in these variables will have a lesser effect on the control of his at very ‘high levels’ or intense feelings of commitment and confidence. With the current data set, we likely will not need to include such an effect to reproduce the individual’s responses since they are bounded \( (C_1, C_2 \in [0, 4]) \).

The individual’s commitment to resist drinking entirely \( C_3(t) \) rarely raises above a level indicating the presence of such a commitment, but on such occasions, it appears to indicate a strong control mechanisms. These occasions occur at times \( t = 25 \) and \( t = 51 \), both of which occur simultaneously with the rare occasions that he does abstain entirely from drinking. While the rate \( a_{17} \) is unknown as are most of the others, it is likely much larger than \( a_{15} \).

### 7.2 Guilt \( \frac{dG}{dt} \) equation

\[
\frac{d}{dt}G(t) = a_{21} \left( \int_{t_1}^{0} A(t+s)ds - A^*(t) \right)
\]

Patient 6009’s responses for guilt, or the perception of the individual’s drinking over the previous day as excessive, exhibit the same dynamics as was seen with patient 6029, so is initially modeled similarly. As with patient 6029, it does not appear that ‘guilt’ \( G(t) \) influences his drinking, but rather the other way around. Thus, his guilt is proportional to his drinking over the past day \( a_{21} \int_{t_1}^{0} A(t+s)ds \) such that it only increases if his drinking has exceeded his personal standard for an ‘acceptable level’, or a threshold \( A^*(t) \). Patient 6009 differs from patient 6029 in that his acceptable level \( A^*(t) \) does not change depending on whether it is a weekend or weekday, but rather is a constant level \( A_1^* \) before his initial pivotal change \( t = \bar{t}_1 \) at a higher level than afterwards, when it is a much lower level \( A_2^* \) (with \( A_1^* > A_2^* \)). Thus, the threshold is represented by

\[
A^*(t) = \begin{cases} 
A_1^* & 0 < t < \bar{t}_1 \\
A_2^* & \bar{t}_1 \leq t \leq T_f 
\end{cases}
\]

While we cannot be sure of the actual day the individual experienced his change \( t = \bar{t}_1 \), we can conclude that it was between the narrow window of \( \bar{t}_1 \in [11, 14] \), or when we see the first significant decrease in drinks. Even more telling is that he reports an increase in feeling that his drinking was excessive at \( t = 14 \) when he drank roughly 6 drinks in a day, as opposed to around 10 drinks the previous time point at \( t = 11 \). This clearly indicates a change in personal standards within this time interval.

### 7.3 Computational approach and results

We constructed piecewise linear functions to best estimate and capture important aspects of data in the supporting variables \( (L(t), C_1(t), C_2(t), C_3(t)) \). These functions were then used as inputs in the solution of the drinking equation \( \frac{d}{dt}A(t) \), and can be seen in Figure 20.
Figure 20: Constructed functions for supporting variables \((L(t), C_1(t), C_2(t), C_3(t))\), used as inputs in the \(\frac{d}{dt} A(t)\) equation for patient 6009.

As with the ‘fitting’ of the model for patient 6029 to his data, the process of working toward a model solution that agreed with patient 6009’s drinking data led to model clarifications. The most important of these was that the nature of the dependence of the equation \(\frac{d}{dt} A(t)\) on the supporting variables is more accurately their time-derivatives, as seen with the relationship between desire and drinking rate with patient 6029. This dependence first became evident when using the loneliness data in the drinking equation. From the data, we hypothesized this dependence as being threshold-dependent (Equation (20)), but as \(L(t)\) increases, while being above the threshold \(L^*\), this induces a decrease in drinking rate. If the threshold \(L^*\) was taken to be at or close to a peak (local maximum) of the loneliness data, the drinking rate would not decrease for a long enough time period to reproduce the decrease as seen in the drinking data. Rather, the increases and decreases of loneliness occur alongside (in time) with of those of drinks. If the input for \(L(t)\) is changed from the piecewise linear function shown in Figure 20 to the slopes of those lines instead, the drinks as predicted by the \(\frac{d}{dt} A(t)\) equation agree much better with the fluctuations seen in the data. Through similar reasoning, when considering the inputs of \(C_3(t), C_1(t), \text{and } C_2(t)\), it became clear that \(\frac{d}{dt} A(t)\) should involve time derivatives of these variables as well. Therefore, the slopes of the piecewise linear functions were used to approximate the data of the supporting variables were used as inputs instead of the piecewise linear functions themselves. Not only do these relationships make more sense in their interpretation, but they gave a model solution that better agrees with the drinking data. The drinks calculated from the model solution and shown on the triweekly time scale along with the triweekly averaged drinking data is shown in Figure 21.

The final model of the drinking data, from which the solution shown in Figure 21 was computed, is given by

\[
\frac{d}{dt} A(t) = a_1(t)\dot{L}(t) - a_2(t)\dot{C}_1(t)\dot{C}_2(t) - a_3(t)\dot{C}_3(t),
\]
where $\dot{X}(t)$ denotes the time derivative of the variable $X$, or $\dot{X}(t) = \frac{d}{dt}X(t)$. The coefficients are time-dependent, so that each piece of the input or supporting variable, is weighted to give a better quantitative fit to the drinking data. For example, the estimated loneliness variable increases from time $t = 0$ to $t = 9$, so the positive slope of that line is taken as $\dot{L}(t)$ and the coefficient $a_1(t)$ is constant over the interval $0 \leq t \leq 9$. After $t = 9$, loneliness decreases until $t = 16$, so that negative slope is then the input for $\dot{L}(t)$ and the coefficient $a_1(t)$ has possible a different constant over the time interval $9 < t \leq 16$. The coefficients and other supporting variables are constructed accordingly as piecewise constant functions.

The need to take the coefficients as time-dependent may be indicative of missing information in the model, but also is likely due to the current limitations in precisely quantifying the supporting variables which do indeed appear to represent dynamic processes. Encouragingly, even if constant coefficients are used (seen in Figure 22), the solution does give the same fluctuations as seen in the data, although the drinks are not as accurately predicted. Namely, the drinks are over-predicted after the initial decrease around time $t = 11$. It is entirely possible that as our ability to quantify the variables improves, there will be less, if at all, of a need to scale with time-dependent coefficients.

Although the guilt data exhibited a similar relationship to drinking data with patient 6009 as with patient 6029, solutions to the initial model equation (21) did not reasonably reproduce dynamics seen in the guilt data. As with the other variables, it appeared that it was due to the dependence of the rate of guilt not being dependent on the drinks over the last day at a given time, but rather, the guilt was directly dependent on the drinks over the last day at a given time. As
Figure 22: The ‘best fit’ model solution, where coefficients $a_1, a_2, a_3$ are constant, shown on the triweekly timescale along with drinking (top) and guilt (bottom) data for patient 6009.

seen in the top right panel of Figure 23, the solution to

$$\frac{d}{dt} G(t) = a_{21} \left( \int_{-\tau}^{0} A(t+s)ds - A^*(t) \right)$$

exhibits the same issues as with the other supporting variables. With drinks above the threshold $A^*(t)$, the guilt increases, although this is not what the data shows. Rather, the data shows a more direct relationship with drinks, and so the bottom two plots are used to illustrate that the relationship seen in the data that guilt is best modeled as being directly proportional to the number of drinks over the past day $G(t) = k \int_{-\tau}^{0} A(t+s)ds$. We note that while mathematically it is also possible to represent drinking as being completely described as proportional to the guilt variable, it does not agree with our interpretation of this patient, and is not as likely true in general of the relationship between guilt and drinking. That is, patient 6009 has not cited guilt from drinking over the previous day as a key contributing reason for drinking. On the other hand, guilt, or one’s reflection on his/her previous day’s drinking, is by nature dependent on the previous day’s drinks. Therefore, for patient 6009, the relationship between guilt and drinking appears to be

$$G(t) = \int_{-1}^{0} A(t+s)ds. \quad (23)$$
Figure 23: The top left shows the model solution for the number of drinks $\int_0^t A(t + s)\,ds$ used to calculate guilt $G(t)$. The top right panel shows guilt $G(t)$ plotted on the triweekly scale as calculated from the model solution (21), along with data. The bottom left panel shows guilt if it is directly proportional to the drinking rate $G(t) = kA(t)$, and the bottom right shows guilt if directly proportional to the number of drinks $G(t) = k \int_{-1}^0 A(t + s)\,ds$. 
7.4 Final 6009 model

Beginning with the case summary as we were motivated to do from the case of 6029, we were able to identify some IVR items which contained dynamics that could explain his drinking behavior, albeit incompletely. As we needed to use these dynamics as inputs, much in the same vein as with the weekend/weekday effect displayed in the desire variable data of 6029, we remark that we are not able to construct a full dynamical system, as these supporting variables do not appear to be influenced by each other. Rather, this is evidence of missing information in the IVR questionnaire, further improvements to which are discussed in the next section. It is likely that if all future modeling efforts begin with case summaries, the insight gained may be limited. That is, it may be that there are insights to be gained from other data not emphasized in the case summary and therefore, perhaps excluded from modeling efforts. For that reason, it is important that the data collection be improved to the extent that inverse problem techniques can be applied to future data sets. In light of that, some suggestions are offered in the next section for data collection, and hopefully the modeling of behavior change will be furthered as a result.

Again, the drinking behavior of patient 6009 appears to be dynamic and has been demonstrated as being influenced by other dynamically changing variables. Patient 6009, as opposed to 6029, appears to change in a much different way, with external events playing a key role. It is noteworthy that variables that we would have previously expected to reflect his change in drinking behavior, regardless of the pattern of change, did not. This suggests that generally there is something missing from the battery of questions in the survey and that it may be that the inclusion of a free form response question in the survey would help to address that. In addition, it is interesting that the desire variable was not closely related to drinking, which is another variable that we would expect to be closely related to drinking regardless of the individual’s motivation. This raises questions concerning which aspects of an individual’s behavior and environment we should focus on to understand behavior change.

7.5 Patient 6029 revisited

![Guilt comparison](image)

Figure 24: Guilt as modeled by the differential equation (19) discussed in Section 5.4 (top), compared with guilt modeled by a direct proportionality to drinks (bottom)

Upon reflecting on our findings with patient 6009, one obvious question is whether the patient 6029’s guilt data may be better explained by a direct proportionality relationship. A comparison from the final model in Section 5.4 and this direct proportionality can be found in Figure 24. While both solutions predict the drinking episodes with roughly the same accuracy, the solution
from the differential equation generally increases with time (as can be seen when focusing on the periods of abstaining from drinking), and that of the direct proportionality relationship does not. This suggests that the simpler form (proportionality) is indeed a more accurate explanation of the relationship between drinking and guilt, or question in the IVR of whether or not the individual feels their drinking was excessive. This begs the question as to the true quantitative nature of the relationships inquired about in the IVR data, as discussed further in the last part of Section 8.
8 Improvements to data

Through the process of developing mathematical models for these two initial patients, some issues in data collection have become clear. Some of these are addressed more easily than others, and we do our best to offer solutions where possible in this section. However, some points we discuss here with the intention of bringing to light more general and likely persistent challenges that complicate the application of mathematical methods to study questions in the behavioral sciences.

![Figure 25: Drinks and model variables of patient 6009’s drinking behavior.](image)

![Figure 26: Drinks and model variables of patient 6009’s drinking behavior.](image)

We advise without hesitation that it is better to avoid averaging of information. While it was intuitive that averaging longitudinal trends from different individuals together would result in the loss of information, our initial approach of averaging responses to similar IVR questions for one individual initially seemed appropriate. However, after constructing the categorical models, it became clear that distinctive longitudinal patterns were only seen in data categories based on
few questions, and most variables that were chosen were based on one IVR question. In the case of patient 6029 the desire data, originally the average of three questions, was the only variable based on more than one IVR question. However, when difficulties arose working toward agreement between model solutions and these data, we examined the longitudinal patterns of each question response individually compared with the average (shown in Figure 25). There appears to be a loss of information in that the average response does not reflect general trends among the other statements. More troubling is that there does not appear to be a general trend among the statements, and the responses to each statement change in different ways in time. This indicates that these items are not equivalent or indicative of the same idea, as previously thought. In fact, comparing two seemingly opposite statements (‘I don’t feel like drinking’ and ‘The idea of drinking is appealing’), one would expect perfectly opposite responses, although that is not the case as seen in Figure 26. In light of these observations, we recommend that only one survey question be asked in the future with one clear purpose.

While we can use single statements in the future to inquire about a single concept, understanding why the responses to the statements shown in Figure 26 do not follow essentially an opposite pattern, is more difficult. One possibility is that the patient may have interpreted the meanings of the statements as having slightly different meanings than the intended one. The clinical staff noted that in some cases, the patients did seem slightly confused as to what exactly the question was asking. This can be addressed by a period in which the participants in the study are trained on the meanings of the questions, analogous to instrument calibration in other sciences. That is, the goal is to fine-tune the instrument, in this case the study participant, so that the measurements are accurate and reflect the measured process. The issue still may be a problem simply due to human error, but hopefully could be diminished.

One of the first aspects of developing any model is to determine the appropriate framework based on the quantitative nature of the state variables, and how they change with time. While the data in the IVR is discrete, we think that many of the measured processes are more accurately continuous as many questions inquire about the intensity of feelings. This could be improved by simply allowing for more responses (ranking on a scale from 0 to 9 if via telephone or even more refined responses via a computer where real numbers in contrast to integers are possible). Another improvement would be to frame the questions in a more quantitative way, asking the patients to qualify how much or to what extent they experience feelings, or agree with statements, etc. This is in contrast to the current IVR items which inquired, for example, if the patient agrees with a statement and the most possible responses were ‘definitely false’, ‘false’, ‘neither true nor false’, ‘true’, and ‘definitely true’.

With such few possible responses, being unable to report intensities more or less than the available options is an obvious limitation, and is known as censoring. For example, an individual could report having no confidence in their ability to resist drinking heavily one day, and could be even less confident the next day, but would not be able to report this decrease. One solution is to include a follow-up question as to whether their true response would be off the possible scale of responses, either above or below. If so, then there are statistical techniques, such as expectation maximization (the EM algorithm), that would allow us to estimate the censored data point. This is a frequently used technique when fitting dynamic models to censored data in using inverse problem methodology (e.g., [2, 3, 13]) with data truncated due to measurement limitation either at high or low levels of detections.

Another aspect of observation error is consistency over time. This is an issue virtually universal in all experimental setups, in which the experimenter either calibrates the instrument at each time point and/or includes some control, so that the measurement taken on a given day can be directly compared with those taken on previous days. That is, if something weighs 24.8 grams on one day
and 24.6 grams the next day, we wish to have some confidence that the difference in measurements accurately reflects a difference in mass. This is complicated in the current application as it may be difficult for individuals to accurately rank their feelings or experiences through time. It may be possible to reduce this error by informing the individual of their previous responses, thereby allowing them to reflect on their experiences, feelings, etc. at the previous time(s) and relate that to their current state.

With a fixed set of questions, it is entirely possible that an important event (either external or internal) is missed that is affecting the person's behavior in a crucial way. We suspect this to be the case with patient 6009, since generally the expected variables (confidence and commitment to limit drinking, guilt, desire, etc.) do not clearly explain his reduction in drinking. It may be due to their being no account for the major external influences in his life in the current data set. For example, all 3 control variables peak around $t = 25$ as well as loneliness, and the individual refrains from drinking at this point. This is a striking feature of the data and suggests that there may be something significant happening with this individual, but the current data collection process does not capture it. This also happens in the biological and physical sciences, (example: not measuring glucagon in understanding glucose-insulin balance in bloodstream) but it is more challenging for this system as many of the key players are unclear. The inclusion of a free form response, or question in which the individual is asked to reflect on their day and to specifically speak to anything he/she feels or that might be going on that was not included in the questions, could be advantageous in this regard. This would have allowed us to at least understand what the data was missing, after collection, in this case. In the future, it may be even possible to then tailor the questionnaire to the patient, or to call the patient into a clinic for more intense observation.

The observation that the selected variables with patient 6009 and the desire variable with patient 6029 were actually related to the drinking rate $A(t)$ directly, and therefore, their time derivatives $\frac{d}{dt}$ related, brings to light another challenge in our ability to select for model variables. Longitudinal trends in each variable are difficult enough to determine given the issues in data collection mentioned above in this section. The task of identifying possible relationships among these variables is complicated by these not easily discernible trends. As a result, we have selected variables that fluctuate directly together and not those that affect their time derivatives, which are more difficult to identify from data. This indicates not only a need for improvement in data collection, so that quantitative longitudinal trends will be clearer, but also, a need for improvement in the approach for constructing mathematical models. In the context of data collection, it may be beneficial to design questions to directly inquire about the increase or decrease of a variable, thereby collecting data directly on their rates of change. Doing so will likely reduce the amount of censored responses that may occur, or possibly abolish it completely. Future studies could investigate the benefit of including such questions in patient surveys.

### 9 Concluding remarks

We began our modeling effort in this monograph by selecting a cohort of 6 participants based on their trajectories of drink reduction. We attempted to use statistical methods to identify dimensions or factors that would serve to reduce our data to inform potential model variables. Next we constructed individual categorical models based on hypothesized causal relationships and attempted to build mathematical models. We began with 6029, and we were able to develop a mathematical model that provided a plausible set of dynamic relationships to explain his drinking. This model fits a self-regulation framework with desire driving up drinking and several empirical supported control variables such as commitment and self-efficacy driving down drinking. Upon attempting to
work toward agreement between model solutions and this patient’s data, we found that a simple model with an input function representing the observed time pattern (weekday/weekend) seen in the patient’s desire data provides a good fit to the drinking data. In developing a model for the next patient 6009, input functions were again used representing the trends seen in the supporting variables. From these, the drinking data for this patient were also reproduced well. Therefore, we offer this work as strong support of mathematical modeling to describe dynamic drinking behavior, and as potential for understanding the mechanisms underlying behavior change.

In the process of developing dynamic models for these two patients, it became clear that to use inverse problem techniques such as parameter estimation was inappropriate at this stage. This is not only because of the initial stages of development of mathematical modeling in this area, but it is also due to the integrity of the data. That is to say, while it is rich in longitudinal information, and it clearly shows that dynamic processes are occurring, the method of quantification leaves room for improvement. This is not uncommon when a data set is collected, as this one was, for a different purpose and only later is mathematical modeling used with the data to gain further insight. Further, the shortcomings of these data are indicative of quantification challenges universal in the psychological sciences, as experimental design and data collection require starkly different approaches than those taken in the physical and more tangible life sciences.

On the other hand, the wealth of information available from automated data collection techniques allows us to observe various individuals as they move through their lives and complex environments, and as a result of dynamic interacting factors, make decisions about their behavior. This is a tremendous advantage to data typically available in other fields as they are are inexpensive and noninvasive and these data sets have much potential for modeling and such inverse problem techniques to improve upon understanding and data collection techniques/experimental design as they can be used to inform the amount, type and form of data to be collected.

Several conclusions from our present efforts will be of substantial assistance in future efforts. First, there can be little doubt of the overall usefulness in using a dynamic modeling approach to aid in understanding of sociological/psychological processes involving mechanisms for behavioral change in alcohol use/abuse. The usual statistical regression methodologies which often involve averaging responses over several individuals or over several similar questions (as was done in the earlier efforts reported on here) do not lead to useful or better understanding of mechanisms of change. Indeed, it is well known in the inverse problem community that averaging data leads (even with a dynamic modeling approach) to loss of detail in understanding of mechanisms. For example, the efforts in [5] illustrate that one should not average data (either across individuals or variates, or across covariates in longitudinal data) as this will diminish the ability to detect refined features involving rates, time dependence of mechanisms, etc., as represented in the data.

Our modeling here focuses on trajectories or dynamic variation in change across time in the important variable or variables of interest and then attempts to relate that to more important “hypothesized” variables that may be influencing drinking in an individual. Developing models in such complex environments is an iterative process [10] of beginning with an initial “hypothesized scenario” (such as our initial model in each case with its many compartments) and investigating the feasibility of this scenario as a representation for the observed processes. This was done by carrying out formal model refinement through consideration of the data and a set of iterative steps involving development of categorical models, then carefully examining the data, and developing models that are simpler to explain the data.

The advantages of such a modeling approach in the context of the present investigation are clear: behavioral change is a complex process that occurs over time and involves interactions of different factors. Change appears not to be linear, either change in drinking or the relationship of drinking to other hypothesized key variables like desire, and may involve cumulative and/or delayed effects.
The relationships between drinking and key variables and the relationship of key variables to each other may change over time in nonlinear ways. This motivates the need for nonlinear, dynamic and possibly non-autonomous systems such as differential equations. Thus, typical methods that begin by aggregating individuals together (creating means, SDs) probably serve to obscure the true nature of what is happening. An obvious example that comes out of our study is that by averaging across individuals we may be missing key “pivotal” or “transformative” events or nonlinear change and what is the immediate influence on and of these critical events. In each case, we do see evidence of these, but the information we collected in these specific data sets is insufficient to completely understand what is happening.

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A IVR Questionnaire

Here we have included the transcript of the IVR questionnaire to which the subjects of Project MOTION called in each day.

Thank you for participating in this study. During this interview, please press the pound or number key once you have entered your answer. If you would like to have the current question repeated at any time, press *7. If you would like to have the previous question repeated, please press *1 at any time. You may repeat a question as many times as you need to. If you would like to pause the survey, press *9. If you accidentally terminate the questionnaire, please call back immediately to continue.

Q1-11: ACTIVITIES SINCE YESTERDAY

For the next 11 questions, I’m going to ask you about things that you’ve done since this time yesterday. For each question, answer using a scale of 0 to 3 where:

0 = no,
1 = yes,
2 = yes - but last night only, or
3 = yes, both last night and today.

(Stressful events)
1) Did you have or nearly have an argument or disagreement with anyone?
2) Did anything else happen at home, work or school that you felt was stressful?
3) Did anything else happen to you that most people would consider stressful?

(Pleasant events)
4) Did you meet a goal or complete a task that left you with a sense of accomplishment?
5) Did you have a pleasant interaction with a family member?
6) Did you have a pleasant interaction with a friend or colleague?

(Encountered Drinking Situation)
7) Did anyone pressure you to drink?
8) Were you in a situation where you commonly drink?
9) Were you at a celebration or a party?
10) Were you at a nightclub or bar?
11) Were you on a date?

**CURRENT MOOD**

Now I’m going to ask you about your mood right now. On a scale from 0 to 4 where:

0 = not at all  
1 = slightly  
2 = moderately  
3 = very much  
4 = extremely  

Right now, do you feel:

1) Active?  
2) Sad?  
3) Nervous?  
4) Tense?  
5) Lonely?  
6) Happy?  
7) Angry?  
8) Enthusiastic?  
9) Bored?  
10) Tranquil?  
11) Relaxed?

**PERCEIVED STRESS**

1) That you are unable to control the important things in your life?  
2) Confident about your ability to handle your personal problems?  
3) That things are going your way?  
4) That difficulties are piling up so high that you can not overcome them?

**YOUR DESIRE TO DRINK RIGHT NOW**

Next I’m going to ask you about your desire to drink right now. Please answer the following questions rating your current desire to drink from 0 to 4, where:

0 = definitely false  
1 = slightly false  
2 = neither true nor false  
3 = slightly true  
4 = definitely true  

Right now:

1) I really don’t feel like drinking.  
2) I feel like I could really use a drink.  
3) The idea of drinking is appealing.

**COMMITMENT AND CONFIDENCE**

On a scale of 0 to 4, where:

0 = not at all  
1 = somewhat  
2 = moderately  
3 = very  
4 = totally
1) How confident are you that you can resist drinking heavily (that is, resist drinking more than 5 drinks) over the next 24 hours?
2) How committed are you not to drink heavily (that is, not to drink more than 5 drinks) over the next 24 hours?
3) How committed are you not to drink AT ALL over the next 24 hours?

DRINKS
Now I would like to ask you some questions about your drinking over the past day. Let’s start with last night’s drinking. Please indicate how many standard drinks you had in each beverage category last night – from the time you took the survey yesterday until you went to sleep.

How many beers did you drink last night? Remember, a standard drink is 12 ounces of beer: _____
0 = no beer

What kind of beer was it mainly?
1 = light beer
2 = regular beer
3 = ale or malt liquor

How many standard drinks of wine did you have last night? Remember, a standard drink is 5 ounces of wine: _____
0 = no wine

How many standard drinks of liquor did you have last night? Remember, a standard drink is 1 1/2 ounces of liquor: _____
0 = no liquor

Now let me ask you about your drinking today. Please indicate how many standard drinks you had in each beverage category today.

How many beers did you drink today? A standard drink is 12 ounces of beer: _____
0 = no beer

What kind of beer was it mainly?
1 = light beer
2 = regular beer
3 = ale or malt liquor

How many standard drinks of wine did you have today? A standard drink is 5 ounces of wine: _____
0 = no wine

How many standard drinks of liquor did you have today? A standard drink is 1 1/2 ounces of liquor: _____
0 = no liquor
ASSESSMENT OF DRINKING AS EXCESSIVE

Do you consider the total amount you have had to drink since this time yesterday to be excessive? That is, was it more than you think you should have had?

0 = definitely not
1 = possibly
2 = probably
3 = definitely

Please press one to exit this survey.

References


