Heat Transfer

• Introduction
  – Practical occurrences, applications, factors affecting heat transfer
  – Categories and modes of heat transfer
• Conduction
  – In a slab and across a pipe
• Convection
  – Free (natural) and forced (in a pipe and over a solid object)
  – Determination of convective heat transfer coefficient (h and h_{f p})
• Radiation
• Thermal resistances to heat transfer
• Overall heat transfer coefficient (U)
• Steady state heat transfer
  – In a tubular heat exchanger (without and with insulation)
• Dimensionless numbers in heat transfer
  – Steady: Reynolds #, Prandtl #, Nusselt #, Grashof #; Unsteady: Fourier #, Biot #
• Unsteady state heat transfer
  – For conduction/convection driven heat transfer; Heisler chart
Practical Occurrences

• Is a metallic park bench colder than a wooden park bench?
• What is wind-chill factor? What is heat index?
• Why dress in layers during winter?
• How does a fan provide cooling effect? Does it blow cold air?
• What is the insulation used in houses? Is it for winter or summer?
• Why does our skin dry-up in a heated room?
• What time of the day and why do we get sea-breeze?
• Why are higher altitude places colder?
• Does hot water freeze faster than cold water?
• In winter, do hot or cold water pipes burst first?
• What is greenhouse effect? What is the principle behind it?
• Can you lose weight by drinking cold water?
• Why are “fins” present on the outside of the radiator of a car?
• “Bridge freezes before road” -- Why?
• Why is salt used to melt ice on the road? When is sand used?
• How does an igloo keep an Eskimo warm?
• Why do you see cars breakdown or pull over to the shoulder of a highway during traffic jams? Do traffic jams cause breakdowns or do breakdowns cause traffic jams?

Heat Transfer in Various Industries

• Automobile: Radiator and engine coolant
• Electronics: Cooling of motherboard/CPU by fan
• Pharmaceutical: Freeze drying of vaccines
• Metallurgical: Heating/cooling during steel manufacture
• Chemical: Condensation, boiling, distillation of chemicals

Home: Refrigerator, AC, heater, dryer, stove, microwave

Heat Transfer in the Food Industry

• Melting: Thawing of a frozen food (turkey)
• Freezing: Freezing of ice-cream mix
• Drying: Drying of fruits
• Evaporation: Spray drying of coffee or concentration of juices
• Sublimation: Freeze drying of coffee
• Heating/cooling of milk
• Baking of bread
• Processing of canned soups (inactivate microorganisms & maximize nutrient content, color/flavor/texture)
What Factors Affect Rate of Heat Transfer?

• Thermal
  – Specific heat ($c_p$, in J/kg K)
    • Measured using Differential scanning calorimeter (DSC)
  – Thermal conductivity (k in W/m K)
    • Measured using Fitch apparatus or thermal conductivity probe (Lab #5)

• Physical
  – Density ($\rho$ in kg/m$^3$)
    • Measured using pycnometer

• Rheological (measured using rheometer/viscometer)
  – Viscosity ($\mu$ in Pa s) for Newtonian fluids
  OR
  – Consistency coefficient ($K$ in Pa s$^n$) and flow behavior index ($n$)
    for power-law fluids

Note: Thermal diffusivity ($\alpha = k/\rho c_p$ in m$^2$/s) combines the effect of several factors.

Specific Heat, Thermal Conductivity, and Thermal Diffusivity

• Specific heat ($c_p$)
  – A measure of how much energy is required to raise the temperature of an object

• Thermal conductivity (k)
  – A measure of how quickly heat gets conducted from one part of an object to another

• Thermal diffusivity ($\alpha$)
  – It combines the effects of specific heat, thermal conductivity, and density of a material. Thus, this one quantity can be used to determine how temperature changes at various points within an object.

Specific Heat (DSC Method)

Heat flux held constant & temperature diff. measured

\[ Q = m_1 \, c_{p(1)} \, (\Delta T_1) = m_2 \, c_{p(2)} \, (\Delta T_2) \]
\[ c_{p(2)} = \left( \frac{m_1}{m_2} \right) \left( \frac{\Delta T_1}{\Delta T_2} \right) \, c_{p(1)} \]

Differential Scanning Calorimeter (DSC)

Manufacturer: Perkin-Elmer
Thermal Conductivity (Fitch Apparatus)

\[ k = \frac{m c_p A}{A t} \ln \left( \frac{T_i - T_x}{T_i - T_x - T_{\infty}} \right) \]

This can be rewritten as:

\[ \text{Slope} = -\frac{kA}{m c_p L} \]

Solve for \( k \):

\[ k = -\left( \text{Slope} \right) \left( \frac{m c_p L}{A} \right) \]

Plot \( \ln \left( \frac{T_i - T_x}{T_i - T_x - T_{\infty}} \right) \) on y-axis versus \( t \) on x-axis & set intercept = 0

Slope = \(-kA/(m c_p L)\); Solve for \( k \):

\[ k = -\left( \text{Slope} \right) \left( \frac{m c_p L}{A} \right) \]

Note: \( k \) is always a positive number

Thermal Conductivity Probe

KD2 Pro Probe (Manufacturer: Decagon Devices)

Single needle probe: Can measure ‘\( k \)’

Dual needle probe: Can measure ‘\( k \)’ and ‘\( \alpha \)’

\[ \alpha = \frac{k}{(\rho c_p)} \]

Values of Thermal Conductivity (\( k \))

- Good conductors of heat have high \( k \) values
  - Cu: 401 W/m K
  - Al: 250 W/m K
  - Fe: 80 W/m K
  - Stainless steel: 16 W/m K

- Insulators have very low (but positive) \( k \) values
  - Paper: 0.05 W/m K
  - Cork, fiberglass: 0.04 W/m K
  - Cotton, styrofoam, expanded polystyrene: 0.03 W/m K
  - Air: 0.024 W/m K (lower \( k \) than insulators!)

- Foods and other materials have intermediate to low \( k \) values
  - Foods: 0.3 to 0.6 W/m K (water: ~0.6 W/m K at room temperature)
  - Glass: 1.05 W/m K; Brick: 0.7 - 1.3 W/m K; Concrete: 0.4 - 1.7 W/m K
  - Plastics (commonly used): 0.15 - 0.6 W/m K

- Thermally conductive plastics may have \( k > 20 \) W/m K
Empirical Correlations

\[ c_p = 4.187 \, (X_w) + 1.549 \, (X_p) + 1.424 \, (X_c) + 1.675 \, (X_f) + 0.837 \, (X_a) \]  
Heldman & Singh, 1981

\[ k = 0.61 \, (X_w) + 0.20 \, (X_p) + 0.205 \, (X_c) + 0.175 \, (X_f) + 0.135 \, (X_a) \]  
Choi & Okos, 1984

w: water,  p: protein,  c: carbohydrates,  f: fat,  a: ash

Effect of Temperature on $k$, $\alpha$, $\rho$, $c_p$)

Questions

Q: When the same heating source is used to heat identical quantities of water and butter, which will be hotter after a certain time?

Ans: Butter; because it has a lower specific heat

Q: In winter, is a metallic park bench colder than a nearby wooden park bench?

Ans: NO. A metallic bench has a higher thermal conductivity and hence conducts heat very well, thereby taking away the heat generated by our body very fast and making us feel colder.
Categories of Heat Transfer

- Steady state
  - Temperatures at all points within the system remain constant over time
  - The temperatures at different locations within the system may be different, but they do not change over time
  - Strictly speaking, steady state conditions are uncommon
    - Conditions are often approximated to be steady state
  - Eg.: Temperature inside a room or refrigerator
- Unsteady state
  - Temperature(s) at one or more points in the system change(s) over time
  - Eg.: Temperature inside a canned food during cooking

Modes of Heat Transfer

- Conduction
  - Translation of vibration of molecules as they acquire thermal energy
  - Occurs in solids, liquids, and gases
    - Heat transfer from hot plate to vessel/pot
    - Heat transfer from surface of turkey to its center
- Convection
  - Fluid currents developed due to temp. differences (within a fluid (liquid/gas) or between a fluid and a solid) or the use of a pump/fan
  - Occurs in liquids and gases
    - Heat transfer from hot vessel/pot to soup in it
- Radiation
  - Emission & absorption of electromagnetic radiation between two surfaces (can occur in vacuum too)
  - Occurs in solids, liquids, and gases
    - Radiation from sun; reflective thermos flask; IR heating of buffet food
**Basics of Conduction**

- Conduction involves the translation of vibration of molecules along a temperature gradient as they acquire thermal energy (mainly analyzed within solids; however, it takes place in liquids and gases also)
  - Actual movement of particles does not occur
- Good conductors of electricity are generally good conductors of heat
- Thermal conductivity \( k \) is used to quantify the ability of a material to conduct heat

**Fourier’s Law of Heat Conduction**

Rate of heat transfer by conduction is given by Fourier’s law of heat conduction as follows:

\[
Q = -kA \frac{\Delta T}{\Delta x}
\]

The negative sign is used to denote/determine the direction of heat transfer (Left to right or right to left)

- \( Q \): Energy transferred per unit time (W)
- \( k \): Thermal conductivity (W/m K); it is a +ve quantity
- \( A \): Area of heat transfer (m²)
- \( \Delta T \): Temperature difference across the ends of solid (K)
- \( \Delta x \): Distance across which heat transfer is taking place (m)
- \( Q/A \): Heat flux (W/m²)

**Temperature Difference Across a Slab**

- Slab: \( Q = kA \frac{\Delta T}{\Delta x} \)
  \[ \Delta T = T_1 - T_2 \]

For the same value of \( Q \) (example: use of a heater on one side of a slab),

- For insulators (low \( k \)), “\( T_1 - T_2 \)” is large
- For good conductors (high \( k \)), “\( T_1 - T_2 \)” is small

For the same value of “\( T_1 - T_2 \)” (example: fixed inside temperature of room and outside air temperature),

- For insulators (low \( k \)), \( Q \) is small
- For good conductors (high \( k \)), \( Q \) is large

Note: \( \Delta x \) and \( A \) are assumed to be the same in all of the above situations
Conduction Across a Slab or Cylinder

- Slab: \( Q = kA \left( \frac{\Delta T}{\Delta x} \right) \)
- Cylinder: \( Q = kA_{lm} \left( \frac{\Delta T}{\Delta r} \right) \)

\( k \): Thermal conductivity (W/m K)
\( A \): Area across which heat transfer is taking place (m²)
\( \Delta T = T_1 - T_2 \): Temperature difference (K)
\( A_{lm} \): Logarithmic mean area (m²)

Note: \( A_{lm} \) comes into play when the area for heat transfer at the two ends across which heat transfer is taking place, is not the same.

Logarithmic Mean Area (\( A_{lm} \))

- Slab: Area for heat transfer is same at both ends
- Cylinder
  - Area at one end (outside) is \( A_o = 2\pi r_o L \)
  - Area at other end (inside) is \( A_i = 2\pi r_i L \)
  - Which area should be used in determining \( Q \)?
  - \( A_{lm} = (A_o - A_i) / \ln (A_o/A_i) = 2\pi L (r_o - r_i) / \ln (r_o/r_i) \)
  - Note: \( A_o > A_{lm} > A_i \)

Logarithmic Mean Temp Diff (\( \Delta T_{lm} \))

Double Tube Heat Exchanger

\( \Delta T \) is NOT constant across the length of tube
\( \Delta T_1 = T_{w(o)} - T_{p(i)} \), \( \Delta T_2 = T_{w(i)} - T_{p(o)} \)
\( \Delta T_{lm} = (\Delta T_1 - \Delta T_2) / \ln (\Delta T_1 / \Delta T_2) \)

Note: \( \Delta T_{lm} \) comes into play when the temperature difference across the two ends where heat transfer is taking place, is not the same.
Convection

Basics of Convection

- It involves transfer of heat by movement of molecules of fluid (liquid or gas) due to
  - Temperatures differences within a fluid or between a fluid and a solid object
  OR
  - An external agency such as a pump or a fan
- Convection is a combination of
  - Diffusion (microscopic/molecular level)
    - Random Brownian motion due to temperature gradient
  - Advection (macroscopic level)
    - Heat is transferred from one place to another by fluid movement

Newton’s Law of Cooling for Convection

Rate of heat transfer by convection (for heating or cooling) is given by Newton’s law of cooling as follows:

\[ Q = h \cdot A \cdot (T_s - T_\infty) \]

- \( Q \): Energy transferred per unit time (W)
- \( h \): Convective heat transfer coefficient -- CHTC (W/m²K)
- \( A \): Surface area available for heat transfer (m²)
- \( \Delta T = T_s - T_\infty \): Temperature difference (K)
- \( T_s \): Surface temperature of solid object (K)
- \( T_\infty \): Free stream (or bulk fluid) temperature of fluid (K)

CHTC (h): Measure of rate of heat transfer by convection; NOT a property; depends on fluid velocity, surface characteristics (shape, size, smoothness), fluid properties (\( \mu, k, \rho, c_p \)).
Categories of Convection

- Free (or natural) convection
  - Does not involve any external agency in causing flow
  - Heat transfer between bottom of vessel and fluid in it
  - Cooling of human body
  - Cooling of radiator fluid in car engine during idling
  - \( h_{\text{air-solid}}: 5-25 \text{ W/m}^2 \text{ K}; h_{\text{water-solid}}: 20-100 \text{ W/m}^2 \text{ K} \)
- Forced convection
  - External agency such as fan/pump causes flow
  - Cooling of radiator fluid in car engine during motion
  - Ice-cream freezer (Blast air)
  - Stirring a pot of soup
  - Heat transferred from computers (fan)
  - \( h_{\text{air-solid}}: 10-200 \text{ W/m}^2 \text{ K}; h_{\text{water-solid}}: 50-10,000 \text{ W/m}^2 \text{ K} \)
  - \( h_{\text{boiling water or steam to solid}}: 3,000-100,000 \text{ W/m}^2 \text{ K} \)

Free Convection

- Fluid comes into contact with hot solid
- Fluid temperature near solid increases
- Fluid density near solid decreases
- This results in a buoyancy force that causes flow
- Rate of heat transfer (\( Q \) & \( h \)) depends on
  - Temperature difference between fluid and surface of solid
  - Properties (\( \mu, \rho, k, c_p \)) of fluid
  - Dimensions and surface characteristics (smoothness) of solid

\[ N_{\text{tu}} = \frac{h_d k}{\nu} = f(N_{\text{Gr}}, N_{\text{Pr}}) \]

Question

Q: What is wind-chill factor? In winter, a thermometer reads -20 °C when air is stationary. All of a sudden, a gust of wind blows. What will the thermometer read?

Ans: -20 °C. As wind speed increases, more heat is removed from our body due to an increase in ‘\( h \)’ and hence ‘\( Q \)’. Thus, we feel colder than when the air is stationary. The air is NOT colder, we just feel colder since more heat is removed from our body and our body is unable to generate enough heat to replace the energy lost to the surroundings.
Nusselt Number ($N_{Nu}$)

$$N_{Nu} = \frac{h d_c}{k_f}$$

- $h$: Convective heat transfer coefficient (W/m² K)
- $d_c$: Characteristic dimension (m)
- $k_f$: Thermal conductivity of fluid (W/m K)

Nusselt number represents the ratio of heat transfer by convection & conduction.

Grashof ($N_{Gr}$) Number

$$N_{Gr} = \frac{\beta_f g \rho_f^2 (T_s - T_\infty) d_c^3}{\mu_f^2}$$

- $\beta_f$: Coefficient of volumetric thermal expansion (K⁻¹)
- $g$: Acceleration due to gravity (= 9.81 m/s²)
- $\rho_f$: Density of fluid (kg/m³)
- $T_s$: Surface temperature of solid object (K)
- $T_\infty$: Free stream temperature of fluid (K)
- $d_c$: Characteristic dimension of solid object (m)
- $\mu_f$: Viscosity of surrounding fluid (Pa s)

Grashof number represents the ratio of buoyancy and viscous forces.

Prandtl Number ($N_{Pr}$)

$$N_{Pr} = \frac{c_{pf} \mu_f}{k_f}$$

- $c_{pf}$: Specific heat of fluid (J/kg K)
- $\mu_f$: Viscosity of fluid (Pa s)
- $k_f$: Thermal conductivity of fluid (W/m K)

Prandtl number represents the ratio of momentum and thermal diffusivities.
Properties of Air

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>f (10^{-5})</th>
<th>h (W/m²K)</th>
<th>0.71</th>
<th>0.72</th>
<th>0.73</th>
<th>0.74</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>185.15</td>
<td>1.61</td>
<td>1.00</td>
<td>0.8256</td>
<td>16.0</td>
<td>16.379</td>
</tr>
<tr>
<td>20</td>
<td>235.15</td>
<td>1.28</td>
<td>0.91</td>
<td>0.8537</td>
<td>19.2</td>
<td>17.496</td>
</tr>
<tr>
<td>30</td>
<td>295.15</td>
<td>1.11</td>
<td>0.91</td>
<td>0.8738</td>
<td>21.5</td>
<td>19.696</td>
</tr>
<tr>
<td>40</td>
<td>365.15</td>
<td>1.09</td>
<td>0.90</td>
<td>0.8908</td>
<td>23.5</td>
<td>20.890</td>
</tr>
<tr>
<td>50</td>
<td>435.15</td>
<td>1.03</td>
<td>0.87</td>
<td>0.9027</td>
<td>25.3</td>
<td>20.935</td>
</tr>
<tr>
<td>60</td>
<td>505.15</td>
<td>1.00</td>
<td>0.85</td>
<td>0.9069</td>
<td>27.3</td>
<td>21.060</td>
</tr>
<tr>
<td>70</td>
<td>575.15</td>
<td>0.97</td>
<td>0.84</td>
<td>0.9069</td>
<td>29.8</td>
<td>21.261</td>
</tr>
</tbody>
</table>

Source: Adapted from Rauhut (1978).

Properties of Water

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>f (10^{-5})</th>
<th>h (W/m²K)</th>
<th>0.71</th>
<th>0.72</th>
<th>0.73</th>
<th>0.74</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>185.15</td>
<td>1.61</td>
<td>1.00</td>
<td>0.8256</td>
<td>16.0</td>
<td>16.379</td>
</tr>
<tr>
<td>2.0</td>
<td>235.15</td>
<td>1.28</td>
<td>0.91</td>
<td>0.8537</td>
<td>19.2</td>
<td>17.496</td>
</tr>
<tr>
<td>3.0</td>
<td>295.15</td>
<td>1.11</td>
<td>0.91</td>
<td>0.8738</td>
<td>21.5</td>
<td>19.696</td>
</tr>
<tr>
<td>4.0</td>
<td>365.15</td>
<td>1.09</td>
<td>0.90</td>
<td>0.8908</td>
<td>23.5</td>
<td>20.890</td>
</tr>
<tr>
<td>5.0</td>
<td>435.15</td>
<td>1.03</td>
<td>0.87</td>
<td>0.9027</td>
<td>25.3</td>
<td>20.935</td>
</tr>
<tr>
<td>6.0</td>
<td>505.15</td>
<td>1.00</td>
<td>0.85</td>
<td>0.9069</td>
<td>27.3</td>
<td>21.060</td>
</tr>
<tr>
<td>7.0</td>
<td>575.15</td>
<td>0.97</td>
<td>0.84</td>
<td>0.9069</td>
<td>29.8</td>
<td>21.261</td>
</tr>
</tbody>
</table>

Free Convection (Plate)

- \( N_{Nu} = \frac{h d_s}{k_f} = f (N_{Gr}, N_{Re}) \)
- \( N_{Nu} = a (N_{Gr} N_{Pr})^{\beta}; \quad N_{Gr} = \frac{N_{Gr}}{N_{Pr}} \)
- For vertical plate (\( d_s = \) plate height)
  - \( a = 0.59, m = 0.250 \) (for \( 10^4 < N_{Re} < 10^6 \))
  - \( a = 0.10, m = 0.333 \) (for \( 10^6 < N_{Re} < 10^{13} \))
- For inclined plate (for \( N_{Re} < 10^9 \))
  - Use same eqn as vertical plate & replace ‘\( g \)’ by ‘\( g \cos \theta \)’ in \( N_{Gr} \)
- For horizontal plate (\( d_s = \) Area/Perimeter)
  - Upper surface hot
    - \( a = 0.54, m = 0.250 \) (for \( 10^4 < N_{Re} < 10^6 \))
    - \( a = 0.15, m = 0.333 \) (for \( 10^6 < N_{Re} < 10^{13} \))
  - Lower surface hot
    - \( a = 0.27, m = 0.250 \) (for \( 10^6 < N_{Re} < 10^{13} \))
Free Convection (Cylinder)

- For vertical cylinder ($d_c = \text{cylinder height}$)
  - Similar to vertical plate if $D \geq 35L/(N_Gr)^{0.25}$

- For horizontal cylinder ($d_c = \text{cylinder diameter}$)
  - For $10^6 < N_{Ra} < 10^{12}$
  
  $Nu = 0.6 + \left[ \frac{0.587 N_{Gr}^{4/5}}{1 + \left( \frac{0.519}{N_{Pr}} \right)^{8/7}} \right]^{1/4}$

  Note: $N_{Ra} = N_{Gr} N_{Pr}$

Free Convection (Sphere)

For sphere, $d_c = \frac{\pi D}{2}$

$Nu = 2 + \frac{0.589 N_{Gr}^{1/4}}{\left[ 1 + \left( \frac{0.469}{N_{Pr}} \right)^{9/4} \right]^{1/4}}$ for $N_{Ra} \leq 10^{11}$ & $N_{Pr} \geq 0.7$

$N_{Ra} = N_{Gr} N_{Pr}$

Forced Convection

- Fluid is forced to move by an external force (pump/fan)
- Rate of heat transfer ($Q$ & $h$) depends on
  - Properties ($\mu$, $\rho$, $k$, $c_p$) of fluid
  - Dimensions and surface characteristics (smoothness) of solid
- ‘$h$’ does NOT depend on
  - Temperature difference between fluid and surface of solid
- ‘$h$’ strongly depends on Reynolds number
  - When all system and product parameters are kept constant, it is flow rate (a process parameter) that strongly affects ‘$h$’

$Nu = \frac{hd_f}{k_f} = f(N_{Re}, N_{Pr})$
Categories of Convective Heat Transfer Coefficient for Forced Convection

• Between a moving fluid and a stationary solid object
  – Transfer of heat from hot pipe to a fluid flowing in a pipe
  – Generally depicted by ‘h’

• Between a moving fluid and a moving particle
  – Transfer of heat from a hot fluid to a freely flowing particle in a suspension (particulate/multiphase food)
  – Generally depicted by ‘hfp’

Forced Convection in a Pipe

• \( N_{Re} = \frac{bd_v \rho}{\mu} = f(N_{Re}, N_p) \)
• Three sub-categories of forced convection exist…..
• 1. Laminar flow (\( N_{Re} < 2100 \))
  – A. Constant surface temperature of pipe
    • \( N_{Nu} = 3.66 \) (for fully developed conditions)
  – B. Constant surface heat flux
    • \( N_{Nu} = 4.36 \) (for fully developed conditions)
  – C. Other situations (for entry region & fully developed)
    • \( N_{Nu} = 1.86 (N_{Re} x N_p x d_v/L)^{0.33} (\mu_b/\mu_w)^{0.14} \)

• 2. Transitional flow
  (2100 < \( N_{Re} < 4000 \))
  – Friction factor (f)
    • For smooth pipes:
      \[ f = 0.79 \ln (N_{Re} - 1.62) \]
    • For non-smooth pipes, use Moody chart (graph of: f, \( N_{Re} \), \( \varepsilon/D \))

Moody Diagram

\( \varepsilon = 259 \times 10^{-6} \text{ m for cast iron}; 1.5235 \times 10^{-6} \text{ m for drawn tubing}; 152 \times 10^{-6} \text{ m for galvanized iron}; 45.7 \times 10^{-6} \text{ m for steel or wrought iron} \)
Forced Convection in a Pipe (contd.)

3. Turbulent flow ($N_{Re} > 4000$) of a Newtonian fluid in a pipe,

$$N_{Nu} = 0.023 \left( N_{Re} \right)^{0.8} \left( N_{Pr} \right)^{0.33} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

$\mu_b$: Viscosity of fluid based on bulk fluid temperature
$\mu_w$: Viscosity of fluid based on wall temperature

The term "($\mu_b/\mu_w$)" is called the viscosity correction factor and can be approximated to "1.0" in the absence of information on wall temperature.

Note: For flow in an annulus, use same eqn with $d_i = 4 \left( A_{cs}/W_p \right) = d_{oi} - d_{oi}$

$d_{oi}$: Inside diameter of outside pipe
$d_{ci}$: Outside diameter of inner pipe

Note: For all forced convection situations, use bulk temperature of fluid to determine properties (unless otherwise specified)

‘$h_{fp}$’ for Forced Convection over a Sphere

$$N_{Nu} = \frac{h_{fp}}{k_f} = f \left( N_{Re}, N_{Pr} \right)$$

$N_{Nu} = 2 + 0.6 \left( N_{Re} \right)^{0.5} \left( N_{Pr} \right)^{0.33}$

For $1 < N_{Re} < 70,000$ and $0.6 < N_{Pr} < 400$

Note 1: $d_i$ is the outside diameter of the sphere

Note 2: Determine all properties at the film temperature

$T_{film} = \left( T_s + T_\infty \right)/2$

Comparison of Free and Forced Convection

- Free convection [$Q = hA \Delta T; N_{Nu} = \frac{h_{fp}}{k_f} = f \left( N_{Gr}, N_{Pr} \right)$]
  - Does not involve any external agency in causing flow
  - Temperature difference ($\Delta T$) causes density difference; this causes flow
  - $Q$ & $h$ depend on
    - Temperature difference between surface of solid and surrounding fluid ($\Delta T$)
    - Properties ($\rho, \kappa, c_p$) of fluid
    - Dimensions and surface characteristics (smoothness) of solid

- Forced convection [$Q = hA \Delta T; N_{Nu} = \frac{h_{fp}}{k_f} = f \left( N_{Re}, N_{Pr} \right)$]
  - External agency such as fan/pump causes flow
  - $Q$ & $h$ depend on
    - Properties ($\rho, \kappa, c_p$) of fluid
    - Dimensions and surface characteristics (smoothness) of solid
  - Only "$Q$" and NOT "$h$" depends on temperature difference between surface of solid and surrounding fluid ($\Delta T$)
Radiation

Basics of Radiation Heat Transfer

Rate of heat transfer by radiation is given by Stefan-Boltzmann law as follows:

\[ Q = \sigma A \varepsilon T^4 \]

- \( Q \): Energy transferred per unit time (W)
- \( \sigma \): Stefan-Boltzmann constant (\( = 5.669 \times 10^{-8} \text{ W/m}^2\text{ K}^4 \))
- \( A \): Surface area of object (m²)
- \( \varepsilon \): Emissivity of surface (ranges from 0 to 1.0)
- \( T \): Temperature (K)

Infrared Thermometer

- Infrared thermometer can be used to non-invasively and remotely determine the surface temperature of an object
- Care should be exercised in ensuring that ONLY emitted energy is measured and NOT reflected energy (may have to use non-reflecting tape on metallic surfaces)
- The emissivity of some infrared thermometers can be adjusted; for others, a pre-set value of 0.95 is commonly programmed
Thermal Resistances to Heat Transfer

- Conduction
  - Slab: $Q = kA (\Delta T/\Delta x) = \Delta T/[(\Delta x/kA)]$
  - Cylinder: $Q = kA_{im} (\Delta T/\Delta r) = \Delta T/[(\Delta r/kA_{im})]$
  - Driving force for heat transfer: $\Delta T$
  - Thermal resistance to heat transfer: $(\Delta x/kA)$ or $(\Delta r/kA_{im})$

- Convection
  - $Q = hA (\Delta T) = \Delta T/[(1/hA)]$
  - Driving force for heat transfer: $\Delta T$
  - Thermal resistance to heat transfer: $(1/hA)$

Units of thermal resistance to heat transfer: K/W

Thermal Resistances

Conduction: $Q = kA \Delta T/\Delta x$
Single slab: $Q = \Delta T/[(\Delta x/kA)]$
Multiple slabs: $Q = \Delta T/[(\Delta x/k_{\text{avg}}A) + (\Delta x/k_2A) + \ldots]$
Cylindrical shell: $Q = \Delta T/[(\Delta r/kA_{im})]$
Multiple cylindrical shells: $Q = \Delta T/[(\Delta r/k_{im1}A_{im1}) + (\Delta r/k_{im2}A_{im2}) + \ldots]$

Convection: $Q = hA \Delta T$
Single convection: $Q = \Delta T/[(1/hA)]$
Multiple convections: $Q = \Delta T/[(1/h_{1A_1}) + (1/h_{2A_2}) + \ldots]$

Combination of conduction and convection
Multiple slabs
$Q = \Delta T/[(\Delta x/k_{im1}A_{im1}) + (\Delta x/k_{im2}A_{im2}) + (\Delta x/k_{im3}A_{im3}) + \ldots]$

Multiple cylindrical shells
$Q = \Delta T/[(1/h_{im1}A_{im1}) + (1/h_{im2}A_{im2}) + (1/h_{im3}A_{im3}) + \ldots]$

Units of thermal resistance to heat transfer: K/W
Electrical Analogy

A current (I) flows because there is a driving force, the potential difference (V), across the resistance (R)

\[ I = \frac{V}{R} \]

Q = \frac{\Delta T}{(\text{Thermal Resistance})}

I = \frac{V}{(R_1 + R_2)}

Q = \frac{\Delta T}{(\text{Sum of Thermal Resistances})}

Thermal resistances are additive (similar to electrical resistances)

k, h, U, Resistances, and Temperatures

- As thermal conductivity (k) increases, thermal resistance due to conduction (\(\Delta x/kA\)) decreases
  - Thus, temperature difference between center and surface of object decreases
- As convective heat transfer coefficient (h) increases, thermal resistance due to convection (1/hA) decreases
  - Thus, temperature difference between the fluid and surface of the solid object decreases

Overall Heat Transfer Coefficient (OHTC)
What is Overall Heat Transfer Coefficient?

- OHTC (denoted by the symbol ‘U’) refers to a single quantity that can be used to quantify the effect of all forms (conduction and convection) of heat transfer taking place in a system.
- It facilitates the use of one equation (instead of individual equations for each conductive and convective heat transfer in the system) to determine the total heat transfer taking place in the system.
  - All thermal resistances in the system are added in order to facilitate this process.

OHTC (or U) in Different Scenarios

- Three conductive heat transfers
  \[ \frac{1}{UA} = \frac{\Delta x_1}{k_1A} + \frac{\Delta x_2}{k_2A} + \frac{\Delta x_3}{k_3A} \]
- Two convective heat transfers
  \[ \frac{1}{UA} = \frac{1}{h_1A_1} + \frac{1}{h_2A_2} \]
- One conductive and one convective heat transfer
  \[ \frac{1}{UA} = \frac{\Delta x_1}{k_1A} + \frac{1}{hA} \]

  - Note 1: \( \frac{1}{UA} > \frac{1}{hA} \); Thus, \( U < h \)
  - Note 2: If there is no conductive resistance, \( U = h \)

U: Overall heat transfer coefficient (W/m² K)

“1/UA”: Overall thermal resistance (K/W)

k, h, U, Resistances, and Temperatures

- As thermal conductivity (k) increases, thermal resistance due to conduction (\( \Delta x/kA \)) decreases.
  - Thus, temperature difference between center and surface of object decreases.
- As convective heat transfer coefficient (h) increases, thermal resistance due to convection (1/hA) decreases.
  - Thus, temperature difference between the fluid and surface of the solid object decreases.
- As overall heat transfer coefficient (U) increases, overall thermal resistance (1/UA) decreases.
  - Thus, temperature difference between the two points across which heat transfer is taking place, decreases.

Thermal conductivity: W/m K
Convective heat transfer coefficient: W/m² K
Overall heat transfer coefficient: W/m² K
Thermal resistance to heat transfer: K/W
**Why Dress in “Layers” in Winter?**

- Single jacket of thickness $\Delta x$:
  
  $$Q = \Delta T \frac{1}{(\Delta x/kA)}$$

- Two jackets, each of thickness $\Delta x/2$:
  
  $$Q = \Delta T \left[ \left( \frac{\Delta x}{2kA} \right) + \left( \frac{\Delta x}{2kA} \right) \right] = \Delta T \left[ \frac{\Delta x}{kA} \right]$$

- If the above two expressions are identical, why is it better to wear two jackets, each of thickness $\Delta x/2$?
  
  Air trapped between the two jackets adds a convective thermal resistance. Thus,
  
  $$Q = \Delta T \left[ \left( \frac{\Delta x}{2kA} \right) + \left( \frac{\Delta x}{2kA} \right) + \left( \frac{1}{hA} \right) \right]$$
  
  Thus, total thermal resistance increases and $Q$ decreases.

---

**Effect of Resistance on Temperature**

- A heater is used to maintain the left end of the slab at 95 °C
- Ambient air on the right side is at 5 °C
- What factors determine the magnitude of temperature at right end of slab?

- $T$ is affected by 95 °C at left AND 5 °C at right
- Resistance to heat transfer from left (by conduction) is $\Delta x/kA$
- Resistance to heat transfer from right (by convection) is $1/hA$

- If both resistances are equal, $T = (95 + 5)/2 = 50 °C$
- If conductive resistance is less (occurs when ‘$k$’ is high), $T > 50 °C$
- If convective resistance is less (occurs when ‘$h$’ is high), $T < 50 °C$

**Note:** The same $Q$ flows through the slab and outside

Thus, $Q = kA (95 - T)/\Delta x + hA (T - 5)$
Types of Heating Equipment

- Direct contact
  - Steam injection, steam infusion
- Indirect contact (Other than plate, tubular, Shell & tube, SSHE)
  - Retorts (Using hot water, steam, or steam-air for heating)
    - Batch (Agitation: None, axial or end-over-end): With or w/o basket/crate
    - Continuous (With agitation): Conventional, Hydrostatic
  - Plate: Series, parallel, series-parallel
  - Tubular: Double tube, triple tube, multi-tube
  - Shell & tube: Single pass, multiple pass, cross-flow
  - Scraped surface heat exchanger (SSHE)
- Alternative/Novel/Emerging Technologies
  - Microwave and radio frequency heating
    - Uses electromagnetic radiation; polar molecules heat up
  - Ohmic heating
    - Electric current in food causes heating; ions in food, cause heating

Steam Injection

Intense, turbulent mixing of steam and product occurs. It results in rapid heating and dilution of product. A vacuum chamber is used downstream to evaporate steam that condensed into product.

Steam Infusion

This is a gentler process than steam injection. A vacuum chamber is used downstream to evaporate steam that condensed into product.
Batch Retorts
- Static (with or without crates/baskets)
  - Horizontal
  - Vertical
- Rotary (axial or end-over-end rotation)
- Reciprocating
  - Shaka

Rotary, end-over-end or reciprocating motion is used to mix the product and make the temperature distribution uniform.

Static & Rotary Retorts
- Horizontal Basket Retort
- SuperAgri Retort

Crateless Retort (Semi-Continuous)

Problem: Cold spot identification
Conventional Continuous Rotary Retort

Hydrostatic Retort

Height of water column provides enough pressure to prevent water from boiling at temperatures well above 100 °C

Hydrostatic Retort/Sterilizer
Plate Heat Exchanger (PHE)

- Simple, efficient, inexpensive; used for not too viscous fluids
- Hot fluid flows on one side of plate and cold fluid flows on other side of plate. Heat transfer occurs across each plate.

PHEs (All channels in Parallel)

- Advantage: Low pressure drop
- Disadvantage: Low heat transfer

PHEs (All Channels in Series)

- Advantage: High heat transfer
- Disadvantage: High pressure drop
PHEs (Series & Parallel)

Advantage
Optimized pressure drop and heat transfer

2 x 2
1 x 4

Regeneration in a PHE

Regeneration: Energy of hot pasteurized product is used to pre-heat cold raw product. Typical regeneration efficiency is ~90%.

Double Tube, Triple Tube, Multitube HX

Heating from 1 side

Heating from 2 sides
Shell & Tube: (One & Two Pass)

Note: Presence of baffles creates cross-flow pattern (product and heating/cooling medium flow at right angles to one another) with uniform heat transfer throughout the heat exchanger. Baffles prevent short-circuiting of heating medium directly from the inlet port to the outlet port.

Scraped Surface Heat Exchanger (SSHE)

Advantage: Mixing of viscous foods
Disadvantage: Particle damage, uncertain residence time, cleaning

Double Tube Heat Exchanger (DTHE)

Double tube heat exchanger
Two concentric tubes
Product generally flows in inner tube
Heating/cooling medium generally flows in outer tube (annulus)
One stream gains heat while the other stream loses heat (hence, heat exchanger)
Both streams may flow in the same or opposite directions
Heat Transfer in a Double Tube HX

(Heat water heating a product)

\[ Q = h_o A_o \left( T_{\text{hot water}} - T_{\text{wall (outside)}} \right) \]

\[ Q = k A_{lm} \left( T_{\text{wall (outside)}} - T_{\text{wall (inside)}} \right) / \Delta r \]

\[ Q = h_i A_i \left( T_{\text{wall (inside)}} - T_{\text{product}} \right) \]

Subscripts for T: 'c' for cold, 'h' for hot, 'i' for inlet, 'o' for outlet

Heat Transfer from Hot Water to Product

- Convection
  - From hot water to outside surface of inner tube
    \[ Q = h_o A_o \left( T_{\text{hot water}} - T_{\text{wall (outside)}} \right) \]
  - Conduction
    - From outside of inner tube to inside of inner tube
      \[ Q = k A_{lm} \left( T_{\text{wall (outside)}} - T_{\text{wall (inside)}} \right) / \Delta r \]
    - Convection
      - From inside surface of inner tube to bulk of product
        \[ Q = h_i A_i \left( T_{\text{wall (inside)}} - T_{\text{product}} \right) \]

Resistances to Heat Transfer from Hot Water to Product

\[ \text{Convective resistance (} 1 / h_o A_o \text{)} \]
\[ \text{Conductive resistance (} \Delta r / k A_{lm} \text{)} \]
\[ \text{Convective resistance (} 1 / h_i A_i \text{)} \]
Overall Heat Transfer Coefficient (U)

\[ Q = \frac{\Delta T}{\left[\frac{1}{h_o A_o} + (\Delta r/k_{Alm}) + \frac{1}{h_i A_i}\right]} \]

Thermal resistances have been added

Denominator: Total thermal resistance

\[ \frac{1}{UA_{lm}} = \frac{1}{h_o A_o} + \frac{\Delta r}{k_{Alm}} + \frac{1}{h_i A_i} \]

Thus, \( Q = \frac{UA_{lm} \Delta T_{lm}}{U} \) \( U: \text{W/m}^2 \text{K} \)

\( U \): Accounts for all modes of heat transfer from hot water to product

\( U \) is NOT a property; it is NOT fixed for a HX; it depends on material properties, system dimensions, and process parameters

\[ \text{Determination of } U: \text{ Theoretical Method} \]

\begin{itemize}
  \item \( \frac{1}{UA_{lm}} = \frac{1}{h_o A_o} + \frac{\Delta r}{k_{Alm}} + \frac{1}{h_i A_i} \)
  \item \( h_i \) and \( h_o \) are usually determined using empirical correlations
  \item ‘\( k \)’ is a material property of the tube of HX
  \item \( A_r, A_o, \) and \( A_{lm} \) are determined based on dimensions (length & radii) of heat exchanger tubes
  \item Once \( A_i, A_o, A_{lm}, h_i, h_o, \) and \( k \) are known, \( U \) is calculated using the above equation
\end{itemize}

\[ \text{Determination of } U: \text{ Experimental Method (Hot Water as Heating Medium)} \]

\[ Q = \dot{m}_c c_{pc} \Delta T_c = \dot{m}_h c_{ph} \Delta T_h \]

\[ = UA_{lm} \Delta T_{lm} \]

Assumption: Heat loss = zero

Once the mass flow rates and temperatures of the product and hot water are experimentally determined, \( U \) can be calculated

If there is heat loss,

\[ Q_{\text{lost by hot water}} = Q_{\text{gained by product}} + Q_{\text{lost to outside}} \]
Tubular Heat Exchanger (Co- and Counter-Current)

\[ T_{\text{in}} = \left( \frac{1}{h_o A_o} + \frac{\Delta r/k A_{\text{lm}}} + \frac{1}{h_i A_i} \right) \Delta T_{\text{in}} \quad \text{and} \quad Q = UA_{\text{lm}} \Delta T_{\text{in}} \]

<table>
<thead>
<tr>
<th>Co-Current Arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot water ( T_{\text{in}} ) ( \rightarrow ) ( T_{\text{out}} )</td>
</tr>
<tr>
<td>Product ( T_{\text{in}} ) ( \rightarrow ) ( T_{\text{out}} )</td>
</tr>
<tr>
<td>( L ) \hbar ( \rightarrow ) ( r )</td>
</tr>
</tbody>
</table>

Q1: Energy transferred from heating medium to product \((=\text{energy gained by product})\)
Q2: Energy lost by heating medium \((=\text{energy gained by product and surroundings})\)
Q3: Energy transferred from heating medium to surroundings \((=\text{energy gained by surroundings})\)

Common approximation: Q3 = 0 \((\text{valid if HX is insulated})\)

In this case,

\[ Q_1 = m_c c_p (\Delta T)_c = Q_2 = m_h c_p (\Delta T)_h = UA_{\text{lm}} \Delta T_{\text{in}} \]

with \( U = \frac{1}{h_o A_o} + \frac{\Delta r/k A_{\text{lm}}} + \frac{1}{h_i A_i} \)

Note 1: \((\Delta T)_c = (T_{\text{co}} - T_{\text{ci}})\)
Note 2: \((\Delta T)_h = (T_{\text{hi}} - T_{\text{ho}})\)

Subscripts for \( m, c, T, \Delta T \):
- \( h \) for hot, \( c \) for cold
- \( i \) for inlet, \( o \) for outlet

Subscripts for \( h, A \):
- \( i \) for inside, \( o \) for outside

<table>
<thead>
<tr>
<th>Counter-Current Arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot water ( T_{\text{in}} ) ( \rightarrow ) ( T_{\text{out}} )</td>
</tr>
<tr>
<td>Product ( T_{\text{in}} ) ( \rightarrow ) ( T_{\text{out}} )</td>
</tr>
<tr>
<td>( L ) \hbar ( \rightarrow ) ( r )</td>
</tr>
</tbody>
</table>

Q1: Energy transferred from heating medium to product \((=\text{energy gained by product})\)
Q2: Energy lost by heating medium \((=\text{energy gained by product and surroundings})\)
Q3: Energy transferred from heating medium to surroundings \((=\text{energy gained by surroundings})\)

Common approximation: Q3 = 0 \((\text{valid if HX is insulated})\)

In this case,

\[ Q_1 = m_c c_p (\Delta T)_c = Q_2 = m_h c_p (\Delta T)_h = UA_{\text{lm}} \Delta T_{\text{in}} \]

with \( U = \frac{1}{h_o A_o} + \frac{\Delta r/k A_{\text{lm}}} + \frac{1}{h_i A_i} \)

Note 1: \((\Delta T)_c = (T_{\text{ci}} - T_{\text{co}})\)
Note 2: \((\Delta T)_h = (T_{\text{ho}} - T_{\text{ci}})\)

Subscripts for \( m, c, T, \Delta T \):
- \( h \) for hot, \( c \) for cold
- \( i \) for inlet, \( o \) for outlet

Subscripts for \( h, A \):
- \( i \) for inside, \( o \) for outside
Insulation

Effect of Insulation

- Air has a lower ‘k’ value than most insulating materials. Why do we use insulation then, to minimize heat loss from a heated pipe?
- Does the addition of insulation around a hot pipe surrounded by a cold fluid always decrease the heat loss from the pipe?
  - Not always!
- Addition of insulation
  - Increases the thermal resistance to heat transfer by conduction \( \Delta x/kA \)
  - Decreases the thermal resistance to heat transfer by convection \( 1/hA \)
  - The net effect may be an increase or decrease in heat loss

Heat Loss (Q) without and with Insulation

Without Insulation

\[
\frac{Q}{\Delta T} = \frac{1}{(1/h_oA_o) + (\Delta r/k_{A_{pipe}}) + (1/h_iA_i)}
\]

\[\Delta r_{pipe} = r_2 - r_1 \text{ and } A_o = 2\pi r_1 L \]
\[A_{pipe} = 2\pi L (r_2 - r_1) / \ln (r_2/r_1)\]

With Insulation

\[
\frac{Q}{\Delta T} = \frac{1}{(1/h_oA'o) + (\Delta r/k_{A_{insulation}}) + (\Delta r/k_{A_{pipe}}) + (1/h_iA_i)}
\]

\[\Delta r_{insulation} = r_3 - r_2 \text{ and } A'o = 2\pi r_3 L \]
\[A_{insulation} = 2\pi L (r_3 - r_2) / \ln (r_3/r_2)\]

Setting \(dQ/dr_3 = 0\), and ensuring that \(d^2Q/dr_3^2\) is -ve, yields \(Q_{max}\)

This happens when \(r_3 = k_{ins}/h_o = r_{critical} = r_c\)

Thus, as insulation is added, heat loss increases till \(r_3 = r_{critical} = r_c\), then it decreases if \(r_3 > r_{critical} = r_c\). Heat loss decreases even if a small amount of insulation is added.
Dimensionless Numbers in Heat Transfer

- Reynolds number \( N_{Re} = \frac{\rho_f \bar{u} d_c}{\mu_f} \)
- Nusselt number \( N_{Nu} = \frac{h d_c}{k_f} \)
- Prandtl number \( N_{Pr} = \frac{c_p(T_f) \mu_f}{k_f} \)
- Grashof number \( N_{Gr} = \frac{\beta_f g \rho_f^2 (T_s - T_a) d_c^3}{\mu_f^2} \)
### Dimensionless Numbers (contd.)

- **Biot number**
  \[ N_{Bi} = \frac{h D_c}{k_s} \]

- **Fourier number**
  \[ N_F = \frac{\alpha_s t}{D_s^2} \]
  \[ \alpha_s = k_s / (\rho c_p) \]
  \[ \alpha_s = \text{Thermal diffusivity (m}^2/\text{s}) \]

#### Reynolds number
Ratio of inertial & viscous forces

#### Nusselt number
Ratio of heat transfer by convection & conduction

#### Prandtl number
Ratio of momentum & thermal diffusivities

#### Grashof number
Ratio of buoyancy & viscous forces

#### Biot number
Ratio of internal & external resistance to heat transfer

#### Fourier number
Ratio of heat conduction & heat storage

**Subscripts**: 'f' for fluid & 's' for solid

- \( D_c \) (for forced convection pipe flow) = 4 (Across-section)/(Wetted perimeter)
- \( D_c \) (for unsteady state heat transfer): Distance between hottest & coldest points in solid object

---

### Nusselt # \( (N_{Nu}) \) and Biot # \( (N_{Bi}) \)

- Both are denoted by \( hd/k \)

- **Nusselt #**
  - Used in STEADY state heat transfer (to determine ‘h’)
  - \( d_c \): Characteristic dimension (= pipe diameter for flow in a pipe)
  - \( k_f \): Thermal conductivity of FLUID

- **Biot #**
  - Used in UNSTEADY state heat transfer (to determine the relative importance of conduction versus convection heat transfer)
  - \( D_c \): Distance between hottest and coldest point in solid object
  - \( k_s \): Thermal conductivity of SOLID

---

### Unsteady State Heat Transfer
(Heat Conduction to the Center of a Solid Object)
Basics of Unsteady State heat Transfer

- Temperature at one or more points in the system changes as a function of time
- Goal of unsteady state heat transfer
  - Determine time taken for an object to attain a certain temperature
  OR
  - Determine temperature of an object after a certain time
- Sometimes, it is used to determine 'h'
- The dimensionless numbers that come into play for unsteady state heat transfer are Biot # (\(N_{Bi} = hD_c/k_s\)) and Fourier # (\(N_{Fo} = \alpha_s t/D_c^2\)) Note: \(\alpha_s = k/(\rho c_p)\)

Categories of Unsteady State Heat Transfer

Categories are based on the magnitude of Biot # (\(N_{Bi} = hD_c/k_s\))

1. Negligible internal (conductive) resistance
   - \(N_{Bi} < 0.1\) (also called lumped capacitance/parameter method)
2. Finite internal and external resistances
   - \(0.1 < N_{Bi} < 40\)
3. Negligible external (convective) resistance
   - \(N_{Bi} > 40\)

\(D_c\) for unsteady state heat transfer: Distance between points of maximum temperature difference within solid object
\(D_s\) for sphere: Radius of sphere
\(D_l\) for an infinitely long cylinder: Radius of cylinder
\(D_t\) for infinite slab with heat transfer from top & bottom: Half thickness of slab
\(D_{ts}\) for an infinite slab with heat transfer from top: Thickness of slab

Modes of Heat Transfer from Air to the Center of a Sphere

- Consider hot air (at 100 °C) being blown over a cold sphere (at 20 °C)

- The two modes of heat transfer are
  - Convection (external) \(Q = hA \Delta T = \Delta T/(1/hA)\)
  - Conduction (internal) \(Q = kA \Delta T/\Delta x = \Delta T/(\Delta x/kA)\)
Significance of Magnitude of $N_B$ (contd.)

$N_B = \frac{hD_s}{k} = \frac{D_s}{k}A$ = Conductive Resistance

$= \frac{1}{h} = \frac{1}{hA}$ = Convective Resistance

- $0.1 < N_B < 40$ (Cat. #2) => Neither conductive nor convective resistance is negligible (both are of the same order of magnitude)
  - Occurs when neither 'h' nor 'k' is very high

Category #1

Temperature Ratio (TR) = $\frac{T - T_\infty}{T_i - T_c} = e^{-\frac{hA}{\rho V c_p}t}$

The above equation can be used to determine temperature, $T$, at time, $t$

Based on time-temperature data, the equation can be used to determine 'h'

$T_i$: Initial temperature of solid object (K)
$T_c$: Temperature of surrounding fluid (K)

$A$: Surface area for heat transfer (m²)
$\rho$: Density of solid object (kg/m³)
$V$: Volume of solid object (m³)
$c_p$: Specific heat of solid object (J/kg K)

$\rho V = \text{mass of object (kg)}$

Shape | Area | Volume
--- | --- | ---
Brick | $2LW + 2WH + LHL$ | $LWH$
Cylinder | $2\pi R L + 2\pi R^2$ | $\pi R^2 L$
Sphere | $4\pi R^3$ | $\frac{4}{3}\pi R^3$

$L$: Length of brick or cylinder
$W, H$: Width, height of brick resp.
$R$: Radius of cylinder or sphere
Category #2

- Need to use Heisler charts
  - TR on y-axis and Fourier number (NFo) on x-axis
  - Several straight lines based on different values of 1/NBi

- Knowing temperature, T (and thus TR), and value of 1/NBi, we can determine x-axis value (or NFo) and hence time, t

OR

- Knowing time, t (and hence NFo or x-axis value), and value of 1/NBi, we can determine y-axis value (or TR) and hence temperature, T

Sample Heisler Chart

Heisler Chart (For Finite Sphere)

Note: \( \alpha_s = k/(\rho c_p) \) = Thermal diffusivity of solid object in m²/s
Category #3

- Need to use Heisler charts to determine time-temperature relation
  - These charts are a way to approximate the exact solution (equation) that represents how temperature (T) changes as a function of time (t)
- Similar approach as category #2
- Since \( N_{Bi} > 40 \), \( 1/N_{Bi} \) is very small (~ 0)
- Thus, we use Heisler charts with the line corresponding to \( 1/N_{Bi} = 0 \)
  - Note: \( 1/N_{Bi} = k/(hD_{c}) \)
### Summary of Categories of Unsteady State Heat Transfer

<table>
<thead>
<tr>
<th>$N_B$</th>
<th>Category 1</th>
<th>Category 2</th>
<th>Category 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_B$</td>
<td>$N_B &lt; 0.1$</td>
<td>$0.1 &lt; N_B &lt; 40$</td>
<td>$N_B &gt; 40$</td>
</tr>
<tr>
<td>This category is encountered when.....</td>
<td>'k' is high OR $D_i$ is small</td>
<td>Neither 'k' nor 'h' are high and $D_i$ is not too small or too large</td>
<td>'h' is high OR $D_i$ is large</td>
</tr>
<tr>
<td>Resistance that is negligible</td>
<td>Conductive (Internal)</td>
<td>None</td>
<td>Convective (External)</td>
</tr>
<tr>
<td>$\Delta T$ that is small</td>
<td>Brown center and surface of solid</td>
<td>None</td>
<td>Brown fluid and surface of solid</td>
</tr>
<tr>
<td>Solution approach</td>
<td>Lumped parameter eqn.</td>
<td>Heisler chart(s)</td>
<td>Heisler chart(s) (with $1/N_B = 0$)</td>
</tr>
</tbody>
</table>

### Heisler Charts

- For a finite sphere
- For an “infininitely” long cylinder
- For a slab “infininitely” long in two dimensions & finite in one dimension

- Heat transfer in one dimension/direction only
- Temperature at ONLY center of object can be determined
  - Use Gurney-Lurie charts for temperatures at other locations within object

*Rule of thumb:* If one dimension of an object is at least 10 times another of its dimension, the first dimension is considered to be “infinite” in comparison to the other.

### Finite Objects

- **Finite brick:** Intersection of 3 infinite slabs
- **Finite cylinder:** Intersection of infinite cylinder & infinite slab
Finite Objects

- Finite objects (such as a cylinder or brick can be obtained as an intersection of infinite objects)

\[
TR_{\text{finite cylinder}} = (TR)_{\text{infinite cylinder}} \times (TR)_{\text{infinite slab}}
\]

\[
TR_{\text{finite brick}} = (TR)_{\text{finite slab width}} \times (TR)_{\text{finite slab width}} \times (TR)_{\text{finite slab height}}
\]

Heisler charts have to be used twice or thrice respectively to determine temperatures for finite cylinder (food in a can) and finite brick (food in a tray)

Note: 
1. \((t)_{\text{finite cylinder}} \neq (t)_{\text{infinite cylinder}} + (t)_{\text{infinite slab}}
2. \((t)_{\text{finite brick}} \neq (t)_{\text{infinite slab width}} + (t)_{\text{infinite slab width}} + (t)_{\text{infinite slab height}}

Calculations for Finite Cylinder (Heisler Chart)

<table>
<thead>
<tr>
<th>Characteristic dimension ((D_c))</th>
<th>Infinite Cylinder</th>
<th>Infinite Slab</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biot number ((N_{Bi})) (N_{Bi} = h D_c / k_s)</td>
<td>If both (N_{Bi} &lt; 0.1), the lumped parameter method (eqn) can be used</td>
<td>(1/N_{Bi})</td>
</tr>
<tr>
<td>Thermal diffusivity (\alpha_s) (\alpha_s = k_s / (\rho_s c_p(s)))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fourier number ((N_{Fo})) (N_{Fo} = \alpha_s D_c^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature ratio ((TR)) from Heisler chart (based on values of (N_{Fo}) &amp; (1/N_{Bi}))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
TR_{\text{finite cylinder}} = (T - T_x) / (T_i - T_x) = (TR)_{\text{infinite cylinder}} \times (TR)_{\text{infinite slab}}\]

Solve for “T” from the above equation

Calculations for Finite Brick (Heisler Chart)

<table>
<thead>
<tr>
<th>Characteristic dimension ((D_c))</th>
<th>Infinite Slab #1</th>
<th>Infinite Slab #2</th>
<th>Infinite Slab #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biot number ((N_{Bi})) (N_{Bi} = h D_c / k_s)</td>
<td>If all 3 (N_{Bi} &lt; 0.1), the lumped parameter method (eqn) can be used</td>
<td>(1/N_{Bi})</td>
<td></td>
</tr>
<tr>
<td>Thermal diffusivity (\alpha_s) (\alpha_s = k_s / (\rho_s c_p(s)))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fourier number ((N_{Fo})) (N_{Fo} = \alpha_s D_c^2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature ratio ((TR)) from Heisler chart (based on values of (N_{Fo}) &amp; (1/N_{Bi}))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
TR_{\text{finite brick}} = (T - T_x) / (T_i - T_x) = (TR)_{\text{infinite slab width}} \times (TR)_{\text{infinite slab width}} \times (TR)_{\text{infinite slab height}}\]

Solve for “T” from the above equation
Temperature Ratio (TR)

\[
TR = \frac{T - T_\infty}{T_i - T_\infty}
\]

- At time \( t = 0 \), \( T = T_i \) and hence \( TR = 1 \)
- At time \( t = \infty \), \( T = T_\infty \) and hence \( TR = 0 \)
- Thus, \( TR \) starts off at 1.0 and can at best reach 0.0

- Low values of \( TR \) (closer to 0.0) indicate a significant change in temperature (from \( T_i \)) of the object
- High values of \( TR \) (closer to 1.0) indicate a minimal change in temperature (from \( T_i \)) of the object

Summary

- Categories of steady state heat transfer
  - Conduction, convection, radiation
- Conduction: Fourier’s law of heat conduction
  - \( Q = -kA \frac{\Delta T}{\Delta x} \); replace \( A \) & \( \Delta T \) by \( A_{lm} \) & \( \Delta T_{lm} \) resp. for cyl.
- Logarithmic mean area
  - \( A_{lm} = \frac{(A_o - A_i)}{\ln (A_o/A_i)} = 2\pi L (r_o - r_i)/[\ln (r_o/r_i)] \)
- Logarithmic mean temperature difference
  - \( \Delta T_{lm} = \frac{(\Delta T_1 - \Delta T_2)}{[\ln (\Delta T_1/\Delta T_2)]} \)
- Convection: Newton’s law of cooling
  - \( Q = hA (\Delta T) \)
- Free convection: Due to density differences within a fluid; Forced convection: Due to external agency (fan/pump)
  - \( Nu = f(NGr & NP_r) \) for free; \( Nu = f(NRe & NP_r) \) for forced

Summary (contd.)

- Thermal resistances to heat transfer by conduction \((\Delta x/kA)\) and convection \((1/hA)\) are additive
- Overall heat transfer coefficient \((U)\) combines the effect of all forms of heat transfer taking place between any two points in a system
- \( Q = UA_{lm} \Delta T_{lm} \) is the most generic form of equation for steady state heat transfer involving conduction and/or convection
- Tubular heat exchanger calculations are based on:
  - \( Q = \dot{m}_p c_{p(p)} \Delta T_{product} = \dot{m}_{hw} c_{p(hw)} \Delta T_{hot \text{ water}} \) (for no heat loss)
  - \( = UA_{lm} \Delta T_{lm} \)
Summary (contd.)

- Thermal properties
  - Specific heat: Important in determining $\Delta T$ of an object
  - Latent heat: Important in determining energy required for phase change
  - Thermal conductivity: Important in determining rate of heat conduction in an object
- For unsteady state heat transfer, the lumped capacitance method (for $N_{Bi} < 0.1$) or Heisler charts (for $N_{Bi} > 0.1$) are used to establish time-temperature relations
  - Heisler charts are applicable only for 1-D heat transfer to determine center temperature
  - Use multiple 1-D objects to create 2-D or 3-D objects
  - Characteristic dimension in unsteady state heat transfer
    - Distance between the hottest and coldest point in object