Mass and Energy Balances
Outline

• Terminology
  – Specific heat, enthalpy, heat, sensible heat, latent heat, saturation temperature & pressure, saturated liquid & vapor

• Phase diagram of water (liquid and vapor phases)
  – Sub-cooled liquid, condensate, steam (saturated steam: mix of condensate & vapor), vapor, superheated steam

• Steam table

• Steam quality

• Superheated steam

• Energy content (or enthalpy)
  – Liquid products, steam

• Mass and energy balances
  – Batch & continuous processes (mixing & heating); direct & indirect contact heating; indirect contact cooling; determination of steam quality
Terminology

• Specific heat ($c_p$: J/kg K)
  – Energy required to raise the temperature of unit mass of a substance by unit value
  – It varies with temperature; however, for approximate calculations, $c_p$ of water is assumed to be 4180 J/kg K

• Enthalpy ($H$: J/kg)
  – Heat content of a system
    • It is always expressed relative to a reference state (usually 0 °C)
      – Generally, enthalpy is assumed to be zero at 0 °C
  – For most liquids, $H \sim c_p (T_{in} \degree C)$

• Heat ($Q$: J)
  – Energy transferred due to temperature difference between system and surroundings
    • When energy transferred is represented per unit time, its units are J/s or W
    • The energy transferred may be utilized as sensible heat or latent heat
Terminology (contd.)

• Sensible heat (Q: J or J/s)
  – Energy transferred from hot to cold object
    • For a batch system: \( Q = m \, c_p \, \Delta T \)  
      \( Q \) is in J
    • For a continuous system: \( Q = \dot{m} \, c_p \, \Delta T \)  
      \( Q \) is in J/s or W

• Latent heat (Q: J or J/s)
  – Energy transferred during phase change without any temperature change
    • For a batch system: \( Q = m \, \lambda \)  
      \( Q \) is in J
    • For a continuous system: \( Q = \dot{m} \, \lambda \)  
      \( Q \) is in J/s or W

• Latent heat of vaporization (\( \lambda_{vap} \): J/kg)
  – Energy reqd to convert 1 kg of a liquid to vapor phase w/o temperature change
  – It equals energy released when vapor condenses to liquid at that temperature
    • For heating foods, the energy of condensing steam is made use of
      – Latent heat of vaporization of water at 100 °C = 2257.06 kJ/kg

• Latent heat of fusion (\( \lambda_{fus} \): J/kg)
  – Energy required to convert 1 kg of a solid to liquid phase w/o temperature change
    • Latent heat of fusion of ice at 0 °C = 333.2 kJ/kg
Terminology (contd.)

- Saturation temperature and pressure
  - Any substance can exist in more than one phase at one time
    - For any pressure, there is a temperature at which a liquid can coexist with the vapor phase
    - This pressure and the corresponding temperature are called the saturation pressure and saturation temperature respectively
    - Example: Water at 101.3 kPa (atmospheric pressure) has a saturation temperature of 100 °C. At higher pressure, saturation temperature is higher.

- Saturated liquid and saturated vapor
  - When only liquid phase exists at the saturation pressure and temperature, the liquid is called saturated liquid or condensate (its enthalpy is denoted by $H_c$)
  - When only vapor exists at the saturation pressure and temperature, the vapor is called saturated vapor (its enthalpy is denoted by $H_v$)
Within the dome, water exists as steam, which is a mixture of liquid and vapor. Here, temperature and pressure are constant and are called the saturation temperature and pressure respectively.

As we move from left to right within the dome, more and more of the water is in vapor phase. The fraction that is in vapor phase is called steam quality (denoted by ‘x’). It varies from 0 to 1 OR 0% to 100%.

Latent heat of vap. ($\lambda_{vap}$) at any temp. or pr. = $H_v - H_c$ at that temp. or pr.
### Saturated Steam Table

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Vapor pressure (kPa)</th>
<th>Specific volume (m³/kg)</th>
<th>Enthalpy (kJ/kg)</th>
<th>Entropy (J/[kg K])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Liquid</td>
<td>Saturated vapor</td>
<td>Liquid</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Saturated vapor</td>
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<tr>
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<td></td>
<td></td>
<td>Liquid</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Saturated vapor</td>
</tr>
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<tr>
<td>150</td>
<td>475.8</td>
<td>0.0010905</td>
<td>0.3928</td>
<td>632.20</td>
</tr>
</tbody>
</table>

|                 |                      |                         |                 | Saturated vapor    |
|                 |                      |                         |                 | Liquid             |
|                 |                      |                         |                 | Saturated vapor    |

|                 |                      |                         |                 | Saturated vapor    |
|                 |                      |                         |                 | Liquid             |
|                 |                      |                         |                 | Saturated vapor    |

What is the latent heat of vaporization of water at atmospheric pressure?

\[ \lambda_{\text{vap}} (\text{at atm. Pr.}) = H_v (\text{at atm Pr.}) - H_c (\text{at atm. Pr.}) = 2676.1 - 419.04 = 2257.06 \text{ kJ/kg} \]
Steam Quality (x)

- The term “Steam” generally refers to saturated steam and not superheated steam
- “Steam” is a mixture of liquid (condensate) and vapor
- The enthalpy of steam (H_s) is a weighted mean of enthalpy of condensate (H_c) and enthalpy of vapor (H_v)
- \( H_s = x \cdot H_v + (1 - x) \cdot H_c \)
- Rearranging, \( x = (H_s - H_c)/(H_v - H_c) \)
- Note that \( x = 0 \) when \( H_s = H_c \)
- Also, \( x = 1 \) when \( H_s = H_v \)
- Higher the steam quality, higher the value of \( H_s \)
- \( 0 \leq x \leq 1 \) OR \( 0\% \leq x \leq 100\% \)

Note: \( H_c \) & \( H_v \) at saturation temperature & pressure are determined from steam tables
Superheated Steam

• When the temperature of vapor is greater than the saturation temperature (at its saturation pressure), the steam is referred to as superheated steam
• The difference between steam temperature and saturation temperature is the degree of superheat
• Superheated steam at a given pressure can have different temperatures greater than the saturation temperature (unlike saturated steam)
• Thus, to define the state of superheated steam, one has to specify the temperature and pressure
  – To define the state of saturated steam one has to specify either temperature or pressure
$Q$ versus $T$

S: Solid (ice)
L: Liquid (water)
V: Vapor (superheated steam)
L+V: Saturated steam

$Q_1 = \lambda_{\text{fusion(ice)}} = 333.2 \text{ kJ/kg}$
$Q_2 = c_p(\text{water}) \Delta T = 419.04 \text{ kJ/kg}$
$Q_3 = \lambda_{\text{vap(water)}} = 2257.06 \text{ kJ/kg}$
Energy Content

• Thermal energy content of a solid or liquid: 
  \[ m \cdot c_p \left( T_{\text{Kelvin}} - 273 \right) \quad \text{OR} \quad m \cdot c_p \left( T_{\text{Celsius}} - 0 \right) \]

• This is based on the assumption that the energy content of the substance is zero at 0 °C (= 273 K)

• Thermal energy content (or enthalpy) of steam is given by: 
  \[ m_s \left( H_s \right) \quad \text{Note: } H_s = x \left( H_v \right) + (1 - x) H_c \]
Heat Exchanger (HX)

• Equipment where heat is exchanged between two products
  – Steam losing heat and cold product gaining heat
  – Hot water losing heat and cold product gaining heat
  – Cold water gaining heat and hot product losing heat

• They may be batch or continuous & direct or indirect contact

• Batch heating: Home microwave oven

• Continuous heating: Pumping a product through a tube and heating it by steam or hot water from outside

• Direct contact heat exchanger
  – Batch heating: Mixing steam with milk in the making of cappuccino
  – Continuous heating: Steam injection/infusion for a flowing product

• Indirect contact heat exchanger
  – Batch heating: Retorts (for canning)
  – Continuous heating: Retorts (canning), double tube, plate, shell & tube
Direct and Indirect Contact Heat Exchanger

• Direct contact heat exchanger
  – Steam contacts product and mixes with it
  – Steam loses energy as it condenses; product gains energy
  – Rapid heat transfer; however, product is diluted
    • Vacuum may be used to remove added water

• Indirect contact heat exchanger (eg. concentric metal pipes)
  – Steam is separated from product by a barrier (metal pipe)
    • Not as much or as rapid heating; however, product is not diluted

• Energy transferred by steam to a product in a direct contact heat exchanger is given by: \( m_s (H_s) \)

• Energy transferred by steam (when steam condenses completely) to a product in an indirect contact heat exchanger is given by: \( m_s (H_s - H_c) \)
Mass Balance

• Overall mass balance
  – For any system, the total mass going into the system must equal the total mass coming out of the system plus any accumulation of mass in the system
    • Batch system: mass \( m \) in kg
    • Continuous system: mass flow rate \( \dot{m} \) in kg/s

• Component mass balance
  – For any component in a system, the total mass of that component going into the system must equal the total mass of that component coming out of the system plus any accumulation of that component in the system

• For any system (or sub-system), the number of mass balance equations = the number of components in the product
  – One mass balance equation can be written for each component
Energy Balance

• Energy balance
  – For any system, the energy going into the system must equal the energy coming out of the system plus any accumulation of energy in the system

• Only ONE energy balance equation is written for any system (or sub-system) irrespective of the number of components in the product(s)

Note: Once mass and energy balance equations are written, make sure that the number of equations and number of unknowns are equal
Applications of Mass and Energy Balance

• Mixing and/or separation of streams of products with different compositions
• Condensation of steam outside a tube through which product flows (indirect contact heating)
• Mixing of steam with a product (direct contact heating)
  – Direct contact heating is faster than indirect contact heating
  – Side-effect of direct contact heating is dilution of product
    • Vacuum may be used to evaporate water that was added
• Evaporation of water from a product to concentrate it

Note: For batch systems, the term “mass” (m) is used and for continuous systems, the term “mass flow rate” (ṁ) is used m: kg  ṁ: kg/s
Mixing of Two Products in a Batch System
(at the same temperature)

The overall mass balance equation is: \( m_1 + m_2 = m_3 \)
The total solids balance equation is: \( m_1 x_1 + m_2 x_2 = m_3 x_3 \)
The total water balance equation is: \( m_1 (1-x_1) + m_2 (1-x_2) = m_3 (1-x_3) \)

The total water balance equation is however NOT a totally new equation. It can be obtained by subtracting the total solids balance equation from the overall mass balance equation. Thus, there are only TWO equations for the above system. Hence, given any 4 variables in the system, the other 2 variables can be determined (Note that the 6 variables in the above system are \( m_1, m_2, m_3, x_1, x_2, x_3 \)). Also note that in a system such as the one shown above, \( x_1 + x_2 \neq x_3 \)
Mixing of Two Products in a Batch System (at different temperatures)

Mixer

\[ m_1, \, x_1 \rightarrow \text{Mixer} \rightarrow \begin{align*} \text{c}_{p(3)}, \, T_3 \\ m_3, \, x_3 \end{align*} \]

\[ m_1, \, m_2, \, m_3: \text{Total masses} \]

\[ x_1, \, x_2, \, x_3: \text{Solid fractions} \]

\[ c_{p(1)}, \, c_{p(2)}, \, c_{p(3)}: \text{Specific heats} \]

\[ T_1, \, T_2, \, T_3: \text{Temperatures} \]

Mass balance equations are the same as before

Energy balance (when temperatures are in Kelvin):

\[ m_1 \, \{c_{p(1)}\} (T_1 - 273) + m_2 \, \{c_{p(2)}\} (T_2 - 273) = m_3 \, \{c_{p(3)}\} (T_3 - 273) \]

Energy balance (when temperatures are in Celsius):

\[ m_1 \, \{c_{p(1)}\} (T_1) + m_2 \, \{c_{p(2)}\} (T_2) = m_3 \, \{c_{p(3)}\} (T_3) \]

\[ c_{p(1)} = x_1 \, c_{p(s)} + (1 - x_1) \, c_{p(w)} \]

\[ c_{p(2)} = x_2 \, c_{p(s)} + (1 - x_2) \, c_{p(w)} \]

\[ c_{p(3)} = x_3 \, c_{p(s)} + (1 - x_3) \, c_{p(w)} \]

\[ c_{p(s)}: \text{Specific heat of solids} \]

\[ c_{p(w)}: \text{Specific heat of water} \]
Direct Heating with Steam (Continuous System)

\[ \dot{m}_f, \ x_f, \ c_{p(f)}, \ T_f \rightarrow \text{Direct contact HX} \rightarrow \dot{m}_p, \ x_p, \ c_{p(p)}, \ T_p \]

Cold product \hspace{2cm} Warm diluted product

Steam \hspace{2cm} \dot{m}_s, \ \text{quality of x}

Pressure, P OR Temperature, T

\( \dot{m}, \ x, \ c_p, \ T: \) Mass flow rate, solids fraction, specific heat, temperature resp.

Subscripts: ‘f’ for feed; ‘p’ for product

**Overall mass balance:** \( \dot{m}_f + \dot{m}_s = \dot{m}_p \)

**Solids balance:** \( \dot{m}_f x_f = \dot{m}_p x_p \)

**Energy balance:** \( \dot{m}_f \left\{ c_{p(f)} \right\} T_f + \dot{m}_s \left( H_s \right) = \dot{m}_p \left\{ c_{p(p)} \right\} T_p \)

\( H_s = (x) H_v + (1 - x) H_c \)

with \( H_v \) and \( H_c \) being determined from steam tables at the steam pressure of P or steam temperature of T
Determination of Steam Quality

$m_s$, quality of $x$; Pressure, $P$ OR Temperature, $T$

Introduce cold water of known mass ($m_c$) and temperature ($T_c$) in a flask
Introduce steam into flask till water becomes hot ($T_h$)
Determine specific heat of water at $T_c$ and $T_h$ from table of properties of water

**Mass balance:** $m_c + m_s = m_h$

**Energy balance:** $m_c \{c_{p(c)}\} T_c + m_s (H_s) = m_h \{c_{p(h)}\} T_h$

Determine $H_s$ from above equation
Also, $H_s = (x) H_v + (1 - x) H_c$
with $H_v$ and $H_c$ being determined from steam tables at the absolute (not gauge) steam pressure of $P$ or steam temperature of $T$
Thus, $x = (H_s - H_c)/(H_v - H_c)$

$m$, $c_p$, $T$: Mass, specific heat, temperature resp.

Subscripts: ‘c’ for cold; ‘h’ for hot

Use interpolation for:
$c_{p(c)}$, $c_{p(h)}$, $H_c$, $H_v$
Indirect Heating with Steam (Continuous System)

\[ \dot{m}_f, \ x_f, \ c_{p(f)}, \ T_f \rightarrow \text{Indirect contact HX} \rightarrow \dot{m}_p, \ x_p, \ c_{p(p)}, \ T_p \]

Cold product \rightarrow \text{Warm undiluted product}

\[ \dot{m}_s, \ \text{quality of x} \]

Pressure, \( P \) \text{ OR } Temperature, \( T \) \text{ 100\% Condensate}

\( \dot{m}, \ x, \ c_p, \ T: \) Mass flow rate, solids fraction, specific heat, temperature resp.

Subscripts: ‘f’ for feed; ‘p’ for product

**Overall mass balance:** \( \dot{m}_f = \dot{m}_p \)

**Solids balance:** \( \dot{m}_f \ x_f = \dot{m}_p \ x_p \) \quad \text{Thus,} \quad x_p = x_f

**Energy balance:** \( \dot{m}_f \ \{c_{p(f)}\} \ T_f + \dot{m}_s \ (H_s) = \dot{m}_p \ \{c_{p(p)}\} \ T_p + \dot{m}_s \ (H_c) \)

\( H_s = (x) \ H_v + (1 - x) \ H_c \)

with \( H_v \) and \( H_c \) being determined from steam tables

at the absolute (not gauge) steam pressure of \( P \) or steam temperature of \( T \)
Indirect Cooling with Water (Continuous System)

\[ \dot{m}_f, \ x_f, \ c_{p(f)}, \ T_f \rightarrow \text{Indirect contact Cooler} \rightarrow \dot{m}_p, \ x_p, \ c_{p(p)}, \ T_p \]

- **Hot feed**

- **Cold water**

- **Cold product**

\[ \dot{m}_f = \dot{m}_p \]

**Solids balance:** \[ \dot{m}_f \ x_f = \dot{m}_p \ x_p \] Thus, \[ x_p = x_f \]

**Energy balance:**
\[ \dot{m}_f \ \{c_{p(f)}\} \ T_f + \dot{m}_{cw} \ \{c_{p(1)}\} \ T_1 = \dot{m}_{cw} \ \{c_{p(2)}\} \ T_2 + \dot{m}_p \ \{c_{p(p)}\} \ T_p \]

- \( c_{p(1)} \) and \( c_{p(2)} \) are determined from tables containing the properties of water at temperatures \( T_1 \) and \( T_2 \) respectively.

When approximations are used, \( c_{p(1)} = c_{p(2)} = 4180 \text{ J/kg K} \)
Summary

- Energy of liquid: \( m \ c_p \ T_{\text{Celsius}} \); Energy of steam: \( m_s \ H_s \)
- Saturated steam tables
  - Enthalpy of condensate (\( H_c \)) and vapor (\( H_v \)) at various saturation T & P
- Steam quality: A measure of the amount of steam in vapor form
  - It ranges from 0.0 to 1.0 OR 0% to 100%
- Calculations for direct heating using steam
  - Overall mass balance
    - Steam is a part of the overall mass balance equation
  - Energy balance
    - No need to factor the energy of steam on RHS of equation
- Calculations for indirect heating using steam
  - Overall mass balance
    - Steam is NOT a part of the overall mass balance equation
  - Energy balance
    - Energy of steam as it exits the system has to be included on RHS of equation